Web Appendix to "Price Discrimination in Input Markets: Downstream Entry and Efficiency"

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B. MULTIPLE FIRMS

With multiple downstream firms of each type, for a given N, the unrestricted uniform wholesale price depends on whether the entrants are more or less efficient than the incumbents. For this reason, in what follows, the unrestricted uniform wholesale price is denoted by $w^u(k_E)$. We first establish that also with multiple firms the unrestricted uniform wholesale price is bracketed by the unrestricted discriminatory wholesale prices, $w^d(k) < w^u(k_E) < w^d(0)$. Since the unrestricted discriminatory wholesale prices are the same irrespective of whether there are one or multiple firms of a particular type, $w^d(k) < w^d(0)$ follows immediately from Lemma 1.

Let n_0 and n_k denote the number of downstream firms with marginal cost 0 and k, respectively. In the absence of entry costs, upstream profits under uniform pricing

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are

$$\Pi^u(w;k) := \begin{cases} n_0\Pi(w;0) + n_k\Pi(w;k) & \text{for} \quad w < P(0) - k \\ n_0\Pi(w;0) & \text{for} \quad P(0) - k \leq w < P(0) \\ 0 & \text{for} \quad w \geq P(0) \end{cases} .$$

Obviously, serving no firm clearly is not optimal. Moreover, under Assumption (A2), it is never optimal to serve only the efficient downstream firms, i.e., we must have $w^u(k_E) < P(0) - k$. Note that $\Pi^u(w;k)$ is strictly concave on $[0, P(0) - k]$. By definition of $w^d(0)$ and $w^d(k)$, $w^d(0) > w^d(k)$, and concavity of $\Pi(w;k_i)$ on $[0, P(0) - k_i]$ for $k_i \in \{0, k\}$, we have

$$\frac{d\Pi^u(w;k)}{dw} = n_0 \frac{d\Pi(w;0)}{dw} + n_k \frac{d\Pi(w;k)}{dw} > 0$$

for all $w \in [0, w^d(k)]$, which immediately implies that $w^d(k) < w^u(k_E)$.

It remains to show that $w^u(k_E) < w^d(0)$. With $w^d(0) < P(0) - k$, under Assumption (A2) we have

$$\frac{d\Pi^u(w^d(0);k)}{dw} = n_0 \frac{d\Pi(w^d(0);0)}{dw} + n_k \frac{d\Pi(w^d(0);k)}{dw} = n_k \frac{d\Pi(w^d(0);k)}{dw} < 0, \quad \text{(B.1)}$$

where the last equality follows from the definition of $w^d(0)$, and the inequality follows from $w^d(0) > w^d(k)$ and $\Pi(w;k)$ being strictly concave on $[0, P(0) - k]$. Strict concavity of $\Pi^u(w;k)$ on $[0, P(0) - k]$ then immediately implies $w^u(k_E) < w^d(0)$.

We next investigate into how $w^u(k_E)$ depends on $N = n_E/n_I$. Implicit differentiation of the equation that implicitly characterizes the unrestricted wholesale price,

$$\begin{aligned} \frac{d\Pi^u(w^u(k_E);k)}{dw} &= n_I \frac{d\Pi(w^u(k_E);k_I)}{dw} + n_E \frac{d\Pi(w^u(k_E);k_E)}{dw} = 0 \\ &\iff \frac{d\Pi(w^u(k_E);k_I)}{dw} + N \frac{d\Pi(w^u(k_E);k_E)}{dw} = 0, \quad \text{(B.2)} \end{aligned}$$

yields

$$\frac{\partial w^u(k_E)}{\partial N} = \frac{\frac{d}{dw}\Pi(w^u(k_E);k_E)}{\frac{d^2}{dw^2}\Pi(w^u(k_E);k_I) + N\frac{d^2}{dw^2}\Pi(w^u(k_E);k_E)}. \quad \text{(B.3)}$$

Since upstream profits from every downstream market are strictly concave on $[0, P(0) - k)$, the nominator is negative. With regard to the numerator, since $w^d(k) <$

$w^u(k_E) < w^d(0) < P(0) - k$, again by concavity of upstream profits we have

$$\frac{d\Pi(w^u(k_E);k)}{dw} < 0 < \frac{d\Pi(w^u(k_E);0)}{dw}. \tag{B.4}$$

This implies that

$$\frac{dw^u(k_E)}{dN} < 0 \quad \text{for} \quad k_E = 0 \qquad \text{and} \qquad \frac{dw^u(k_E)}{dN} > 0 \quad \text{for} \quad k_E = k. \tag{B.5}$$

This insight now allows us to establish how $\bar{F}^u(k_E)$ is affected by a change in N. Implicit differentiation of

$$\pi(w^u(k_E) + k_E) = \bar{F}^u(k_E) \tag{B.6}$$

with respect to N yields

$$\frac{\partial \bar{F}^u(k_E)}{\partial N} \quad = \quad \pi'(w^u(k) + k_E)\frac{dw^u(k_E)}{dN} \begin{cases} < 0 & \text{for} \quad k_E = k \\ > 0 & \text{for} \quad k_E = 0 \end{cases}. \tag{B.7}$$

The threshold $\hat{F}^u(k_E)$ is implicitly defined by

$$\Pi(w^R(\hat{F}^u(k_E);k_E);k_I) + N\Pi(w^R(\hat{F}^u(k_E);k_E);k_E) = \Pi(w^d(k_I);k_I). \tag{B.8}$$

Implicit differentiation of (B.8) with respect to N reveals

$$\frac{\partial \hat{F}^u(k_E)}{\partial N} = -\frac{\Pi(w^R(\hat{F}^u(k_E);k_E);k_E)}{\left(\frac{\partial}{\partial w}\Pi(w^R(\hat{F}^u(k_E);k_E);k_I) + N\frac{\partial}{\partial w}\Pi(w^R(\hat{F}^u(k_E);k_E);k_E)\right)\frac{d}{dF}w^R(\hat{F}^u(k_E);k_E)}. \tag{B.9}$$

Remember that $n_I\Pi(w;k_I)+n_E\Pi(w;k_E)$ is strictly increasing on the interval $[0, w^u(k))$. Since $\hat{F}^u(k_E) > \bar{F}^u(k_E)$ implies $w^R(\hat{F}^u(k_E);k_E) < w^R(\bar{F}^u(k_E);k_E) = w^u(k_E)$, we have $\partial\Pi(w^R(\hat{F}^u(k_E);k_E);k_I)/\partial w + N[\partial\Pi(w^R(\hat{F}^u(k_E);k_E);k_E)/\partial w] > 0$. With the restricted wholesale price being decreasing in F, it follows that

$$\frac{\partial \hat{F}^u(k_E)}{\partial N} > 0. \tag{B.10}$$

C. Downstream Competition with a Less Efficient Entrant

In this appendix, we provide a detailed analysis of the case with downstream Cournot competition as discussed in Section 6. The equilibrium concept employed is subgame perfect Nash equilibrium. We solve the game by backward induction, beginning in stage three.

Stage 3: For given wholesale prices and a given number of active firms in the intermediate industry, we determine the quantities produced of the final good by firms active in the downstream market. If a downstream firm with own marginal cost k_i is a downstream monopolist, its demand for the input at a wholesale price w is

$$q(w+k_i) = \begin{cases} \frac{1-w-k_i}{2} & \text{for} \quad w < 1-k_i \\ 0 & \text{for} \quad w \geq 1-k_i \end{cases} .$$

If two firms i and j are active in the downstream market, then firm i's best response at wholesale price w_i given that firm j produces quantity q_j is

$$q(q_j; w_i + k_i) = \max\left\{0, \frac{1-w_i-k_i-q_j}{2}\right\} \tag{C.1}$$

For $2w_i - w_j < 1-2k_i+k_j$ and $2w_j - w_i < 1-2k_j+k_i$ the Cournot Nash equilibrium is interior with both firms producing strictly positive quantities. The equilibrium quantity of firm $i \neq j$ is

$$q(w_i+k_i, w_j+k_j) = \frac{1-2(w_i+k_i)+(w_j+k_j)}{3}. \tag{C.2}$$

If $2w_i - w_j < 1-2k_i+k_j$ and $2w_j - w_i \geq 1-2k_j+k_i$, then firm j produces nothing whereas firm i produces its monopoly quantity. For $2w_i - w_j \geq 1-2k_i+k_j$ and $2w_j - w_i \geq 1-2k_j+k_i$ both downstream firms produce a zero quantity.

Stage 2 Given wholesale prices w_I and w_E charged from firm I and firm E, respectively, and correctly anticipating Nash equilibrium play in stage three, firm E

enters the market if its profits in the resulting market outcome in stage 3 exceed the entry cost. If indifferent between entering and not entering the market, as a tie-breaking rule we assume that firm E behaves as the manufacturer M wishes.[1] If firm E's profits in stage three are strictly negative, then E does not enter the intermediate industry.

Stage 1 Correctly anticipating firm E's entry decision in stage two and equilibrium play in stage three, M chooses wholesale prices w_I and w_E in order to maximize upstream profits. In what follows, we refer to a duopoly as a situation, in which E enters the downstream market and downstream demand is strictly positive for both firms I and E. Again, when indifferent between implementing a downstream duopoly or a downstream monopoly, the upstream supplier implements a downstream monopoly. Let $\Pi^r_{\{i\}}$ denote M's profit from implementing firm $i \in \{I, E\}$ as a downstream monopolist, and let $\Pi^r_{\{I,E\}}$ denote M's profit from implementing firms I and E as downstream duopolists. Superscript $r \in \{d, u\}$ again refers to either a discriminatory pricing regime or a uniform pricing regime. Moreover, in order not to clutter notation, we will often suppress the dependency of downstream quantity choices on effective marginal costs as well as the dependency of optimal wholesale prices and welfare on the entry cost and own marginal costs of the downstream firms.

Lemma 1: *Under Price discrimination,*

(i) *if* $\sqrt{F} \leq (1/6) - (1/3)k$, *then* M *charges wholesale prices* $w^d_I = w^d(0) = 1/2$ *and* $w^d_E = w^d(k) = (1-k)/2$. *This implements a downstream duopoly resulting in quantities* $q^d_I = (1+k)/6$, $q^d_E = (1-2k)/6$, *and* $Q^d = (2-k)/6$;

(ii) *if* $(1/6) - (1/3)k < \sqrt{F} < (1/3) - (2/3)k$, *then* M *charges wholesale prices* $w^d_I = w^R_I = 1/2$ *and* $w^d_E = w^R_E(\sqrt{F}; k) = (3/4) - k - (3/2)\sqrt{F}$. *This implements a downstream duopoly resulting in quantities* $q^d_I = (1/4) - (1/2)\sqrt{F}$, $q^d_E = \sqrt{F}$, *and* $Q^d = (1/4) + (1/2)\sqrt{F}$;

[1] We impose this alternative tie-breaking rule for expositional purposes only. Sticking to the original tie-breaking rule, i.e., firm E enters whenever its profits are nonnegative, yields exactly the same results.

(iii) if $(1/3) - (2/3)k \leq \sqrt{F}$, *then* M *charges wholesale prices* $w_I^d = w_M = 1/2$ *and* $w_E^d = \infty$. *This implements a downstream monopoly resulting in quantities* $q_I^d = Q^d = 1/4$.

Proof:

Suppose M wants to implement a downstream duopoly. Then M chooses wholesale prices in order to solve the following problem:

Program D-PD:

$$\max_{(w_I, w_E) \in \mathbb{R}^2_{\geq 0}} w_I \frac{1 - 2w_I + (w_E + k)}{3} + w_E \frac{1 - 2(w_E + k) + w_I}{3}$$

$$\begin{aligned} \text{subject to} \quad & q_I = \frac{1 - 2w_I + (w_E + k)}{3} > 0 \\ & q_E = \frac{1 - 2(w_E + k) + w_I}{3} > 0 \\ & F \leq \left[\frac{1 - 2(w_E + k) + w_I}{3} \right]^2 \end{aligned}$$

Next, we show that for a sufficiently low entry cost, the solution to Program D-PD is identical to the solution of the relaxed program, which only considers the latter two constraints.

Claim 1: *If* $\sqrt{F} \leq (1/2) - (2/3)k$, *the solution to Program R*

$$\max_{(w_I, w_E)} w_I \frac{1 - 2w_I + (w_E + k)}{3} + w_E \frac{1 - 2(w_E + k) + w_I}{3}$$

$$\textit{subject to} \quad 2w_E - w_I \leq 1 - 2k - 3\sqrt{F},$$

also solves Program D-PD.

Proof of Claim 1: First, note that the latter two constraints of Program D-PD can equivalently be replaced by the following condition:

$$2w_E - w_I \leq 1 - 2k - 3\sqrt{F}, \tag{C.3}$$

which corresponds to the one constraint in Program R. The Lagrangian associated with Program R is

$$\begin{aligned} \mathcal{L} = {} & w_I \frac{1 - 2w_I + (w_E + k)}{3} + w_E \frac{1 - 2(w_E + k) + w_I}{3} \\ & - \lambda \left\{ 2w_E - w_I - (1 - 2k - 3\sqrt{F}) \right\}. \end{aligned} \tag{C.4}$$

With $\mathcal{L}$ being a strictly concave function, the associated Kuhn-Tucker conditions are sufficient for global optimality. These Kuhn-Tucker conditions are given by

$$\frac{\partial \mathcal{L}}{\partial w_I} = \frac{1 + 2w_E + k - 4w_I}{3} + \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial w_E} = \frac{1 - 4w_E - 2k + 2w_I}{3} - 2\lambda = 0$$
$$\lambda \geq 0 \ \left(= 0 \ \text{ if } \ 2w_E - w_I < 1 - 2k - 3\sqrt{F}\right)$$
$$2w_E - w_I \leq 1 - 2k - 3\sqrt{F}$$

Consider the case of $\sqrt{F} \leq (1/6) - (1/3)k$ first. Suppose the constraint is not binding, i.e., $2w_E - w_I < 1 - 2k - 3\sqrt{F}$. The complementary slackness condition then implies $\lambda = 0$. Combining the two first-order conditions yields wholesale prices $w_I = 1/2$ and $w_E = (1-k)/2$. It is readily verified that for $\sqrt{F} \leq (1/6) - (1/3)k$, at these prices the constraint of Program R is satisfied. Moreover, under these wholesale prices, all remaining constraints of Program D-PD are also satisfied: wholesale prices are nonnegative, and associated quantities are strictly positive, $q_I = (1 + k)/6$ and $q_E = (1 - 2k)/6$. Next, consider the case $(1/6) - (1/3)k < \sqrt{F} \leq (1/2) - (2/3)k$. Suppose that the constraint is binding, i.e., $2w_E - w_I = 1 - 2k - 3\sqrt{F}$. The complementary slackness condition then implies $\lambda \geq 0$. Combining the two first-order conditions yields $w_I = 1/2$. Inserting this into the binding constraint leads to $w_E = (3/4) - k - (3/2)\sqrt{F}$. Solving for the Lagrange parameter yields $\lambda = (-1 + 2k + 6\sqrt{F})/6$, which is strictly positive for $(1/6) - (1/3)k < \sqrt{F}$. It is readily verified that for $\sqrt{F} \leq (1/2) - (2/3)k$ all remaining constraints of Program D-PD are also satisfied under these wholesale prices: wholesale prices are nonnegative, and associated quantities are strictly positive, $q_I = (1/4) - (1/2)\sqrt{F}$ and $q_E = \sqrt{F}$. This proves Claim 1. ||

Straightforward calculations show that M's profit from implementing a downstream duopoly is $\Pi^d_{\{I,E\}} = (1 - k + k^2)/6$ if $\sqrt{F} \leq (1/6) - (1/3)k$, and $\Pi^d_{\{I,E\}} = (1/8) + ((1/2) - k)\sqrt{F} - (3/2)(\sqrt{F})^2$ if $(1/6) - (1/3)k < \sqrt{F} \leq (1/2) - (2/3)k$. Note that for $\sqrt{F} > (1/2) - (2/3)k$, M's problem becomes more heavily constrained, such that M's profit cannot be larger than for $\sqrt{F} \leq (1/2) - (2/3)k$.

Next, suppose M wants to implement a downstream monopoly. When facing a downstream monopolist with own marginal cost k_i, the optimal wholesale price for M to charge is $w = (1 - k_i)/2$, which results in downstream demand $q = (1 - k_i)/4$ and upstream profits $\Pi^d_{\{i\}} = (1 - k_i)^2/8$. Since M's maximum profit decreases in the downstream monopolists own marginal cost, M always prefers I to become a monopolist over E becoming a monopolist. Since under price discrimination M can charge E a prohibitively high price which keeps E out of the downstream market without affecting the price paid by the incumbent firm I, M can always make I the downstream monopolist, resulting in upstream profits of $\Pi^d_{\{I\}} = 1/8$.

In order to conclude the proof of Lemma 2, we have to determine when M prefers to implement a downstream duopoly over implementing a downstream monopoly. If $\sqrt{F} \leq (1/6) - (1/3)k$, $\Pi^d_{\{I,E\}} > \Pi^d_{\{I\}}$ if and only if $(1 - 2k)^2 > 0$. Thus, if $\sqrt{F} \leq (1/6) - (1/3)k$, M will implement a downstream duopoly resulting in quantities $q^d_I = (1+k)/6$ and $q^d_E = (1-2k)/6$. Next, if $(1/6) - (1/3)k < \sqrt{F} \leq (1/2) - (2/3)k$, $\Pi^d_{\{I,E\}} > \Pi^d_{\{I\}}$ if and only if $\sqrt{F} < (1/3) - (2/3)k$. Thus, if $(1/6) - (1/3)k < \sqrt{F} < (1/3) - (2/3)k$, M will implement a downstream duopoly resulting in quantities $q^d_I = (1/4) - (1/2)\sqrt{F}$ and $q^d_E = \sqrt{F}$, whereas for $\sqrt{F} \geq (1/3) - (2/3)k$, M will implement a downstream monopoly resulting in quantity $q^d_I = 1/4$. This establishes the desired result. ■

Lemma 2: *Under uniform pricing,*

(i) if $k < 2 - \sqrt{3}$ and $\sqrt{F} \leq (1/6) - (7/12)k$, then M charges a wholesale prices $w^u = w^u(k) = (1/2) - (1/4)k$. This implements a downstream duopoly resulting in quantities $q^u_I = (2 + 5k)/12$, $q^u_E = (2 - 7k)/12$, and $Q^u = (2 - k)/6$;

(ii) if $k < 2 - \sqrt{3}$ and $(1/6) - (7/12)k < \sqrt{F} < (1/6) - (7/12)k + (\sqrt{1 - 4k + k^2})/12$, then M charges a wholesale prices $w^u = w^{Ru}(\sqrt{F}; k) = 1 - 2k - 3\sqrt{F}$. This implements a downstream duopoly resulting in quantities $q^u_I = k + \sqrt{F}$, $q^u_E = \sqrt{F}$, and $Q^u = k + 2\sqrt{F}$;

(iii) if $k \geq 2 - \sqrt{3}$ *or* $\sqrt{F} \geq (1/6) - (7/12)k + (\sqrt{1 - 4k + k^2})/12$*, then* M *charges a wholesale price* $w^u = w_M = (1/2)$*. This implements a downstream monopoly resulting in quantities* $q_I^u = Q^u = 1/4$*.*

Proof:

Suppose M wants to implement a downstream duopoly. Then M chooses the uniform wholesale price in order to solve the following problem:

PROGRAM D-UNI:

$$\begin{aligned} &\max_{w \in \mathbb{R}_{\geq 0}} w \frac{2 - 2w - k}{3} \\ \text{subject to} \quad & q_I = \frac{1 - w + k}{3} > 0 \\ & q_E = \frac{1 - w - 2k}{3} > 0 \\ & F \leq \left[\frac{1 - w - 2k}{3} \right]^2 \end{aligned}$$

First, note that if the second constraint holds also the first constraint holds with strict inequality, i.e., if E demands a nonnegative quantity at wholesale price w, $q_E \geq 0$, then I demands a strictly positive quantity, $q_I > 0$. Moreover, the second and third constraint together can equivalently be replaced by the following condition: $w \leq 1 - 2k - 3\sqrt{F}$. Thus, Program D-UNI can be equivalently rewritten as

PROGRAM D-UNI:

$$\begin{aligned} &\max_{w \in \mathbb{R}_{\geq 0}} w \frac{2 - 2w - k}{3} \\ \text{subject to} \quad & w \leq 1 - 2k - 3\sqrt{F} \end{aligned}$$

Note that M's objective is maximizing a strictly concave function with a unique global maximum attained at $w = (2 - k)/4$. Therefore, if $(2 - k)/4 \leq 1 - 2k - 3\sqrt{F}$, or, equivalently, if $\sqrt{F} \leq (1/6) - (7/12)k$, the optimal uniform wholesale price that implements a downstream duopoly is $w = (2 - k)/4$, resulting in quantities $q_I = (2 + 5k)/12$ and $q_E = (2 - 7k)/12$. Note that $q_E > 0$—and thus also $q_I > 0$—if and only if $k < 2/7$. If $\sqrt{F} > (1/6) - (7/12)k$, the constraint becomes binding. If $\sqrt{F} \leq (1/3) - (2/3)k$, the optimal uniform wholesale price in order to implement a downstream duopoly is given by $w = 1 - 2k - 3\sqrt{F}$, resulting in quantities $q_I = k + \sqrt{F}$ and $q_E = \sqrt{F}$. If $\sqrt{F} > (1/3) - (2/3)k$, implementation of a downstream duopoly with E demanding a strictly positive quantity and making nonnegative profits is not possible with a nonnegative wholesale price.

Straightforward calculations show that M's profit from implementing a downstream duopoly is $\Pi^u_{\{I,E\}} = (2-k)^2/24$ if $\sqrt{F} \leq (1/6) - (7/12)k$, and $\Pi^u_{\{I,E\}} = (1 - 2k - 3\sqrt{F})(k + 2\sqrt{F})$ if $(1/6) - (7/12)k < \sqrt{F} \leq (1/3) - (2/3)k$.

Next, suppose that M wants to implement a downstream monopoly. As noted above, for a given wholesale price w, if E demands a nonnegative quantity, then I demands a strictly positive quantity. Thus, under uniform pricing, the only possible form monopoly can take in the downstream market is with I as downstream monopolist. Therefore, when implementing a downstream monopoly under uniform pricing, M has to choose a wholesale price at which E does not find it profitable to enter. From above we know that this requires the wholesale price to be sufficiently high, i.e., $w > 1 - 2k - 3\sqrt{F}$. Under our tie-breaking rule that E does what M wants him to do when indifferent between entering and not entering the market, M implements a downstream monopoly whenever he chooses a wholesale price $w \geq 1 - 2k - 3\sqrt{F}$. With the quantity demanded by downstream monopolist I being $q_I = (1-w)/2$, by the choice of the wholesale price M maximizes a strictly concave function with a unique stationary point at $w = 1/2$ subject to the aforementioned constraint. In consequence, if $1/2 \geq 1 - 2k - 3\sqrt{F}$, or equivalently, if $\sqrt{F} \geq (1/6) - (2/3)k$, then the optimal wholesale price to implement a downstream monopoly is $w = 1/2$ resulting in quantity $q_I = 1/4$ and upstream profit $\Pi^u_{\{I\}} = 1/8$. If $\sqrt{F} < (1/6) - (2/3)k$, then the optimal wholesale price to implement a downstream monopoly is $w = 1 - 2k - 3\sqrt{F}$ resulting in quantity $q_I = k + (3/2)\sqrt{F}$ and upstream profit $\Pi^u_{\{I\}} = (1 - 2k - 3\sqrt{F})(k + (3/2)\sqrt{F})$. Note that $w = 1 - 2k - 3\sqrt{F} \geq 0$ if and only if $\sqrt{F} \leq (1/3) - (2/3)k$, which obviously is satisfied for $\sqrt{F} < (1/6) - (2/3)k$.

In order to conclude the proof of Lemma 3, we have to determine when M prefers to implement a downstream duopoly over implementing a downstream monopoly.

Combining the observations obtained above, we have to distinguish four cases. (i) If $\sqrt{F} > (1/3) - (2/3)k$, implementation of a downstream duopoly is not feasible. Thus, M implements an unconstrained downstream monopoly resulting in quantity $q_I^u = 1/8$. (ii) If $(1/6) - (7/12)k < \sqrt{F} \leq (1/3) - (2/3)k$, then $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $(1 - 2k - 3\sqrt{F})(k + 2\sqrt{F}) > 1/8$, or, equivalently, $(\sqrt{F})^2 - ((2-7k)/6)\sqrt{F} + (1 - 8k + 16k^2)/48 < 0$. For $k < 2 - \sqrt{3}$, this condition implies that $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $(1/6) - (7/12)k < \sqrt{F} < (1/6) - (7/12)k + (\sqrt{1 - 4k + k^2})/12$, whereas for $k \geq 2 - \sqrt{3}$ we always have $\Pi^u_{\{I,E\}} \leq \Pi^u_{\{I\}}$. Thus, M implements a downstream duopoly resulting in quantities $q_I^u = k + \sqrt{F}$ and $q_E^u = \sqrt{F}$ if $k < 2 - \sqrt{3}$ and $(1/6) - (7/12)k < \sqrt{F} < (1/6) - (7/12)k + (\sqrt{1 - 4k + k^2})/12$, and a downstream monopoly resulting in quantity $q_I^u = 1/8$ otherwise. (iii) If $(1/6) - (2/3)k < \sqrt{F} \leq (1/6) - (7/12)k$, where the latter inequality implies $k < 2/7$, then $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $(2 - k)^2/24 > 1/8$. This latter condition implies that $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $k < 2 - \sqrt{3}$. Thus, M implements a downstream duopoly resulting in quantities $q_I^u = (2+5k)/12$ and $q_E^u = (2-7k)/12$ if $k < 2 - \sqrt{3}$ and $(1/6) - (2/3)k < \sqrt{F} \leq (1/6) - (7/12)k$, and a downstream monopoly resulting in quantity $q_I^u = 1/8$ otherwise. (iv) If $\sqrt{F} \leq (1/6) - (2/3)k$, which implies $k \leq 1/4$, then $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $(2 - k)^2/24 > (1 - 2k - 3\sqrt{F})(k + (3/2)\sqrt{F})$, or, equivalently, $(\sqrt{F})^2 + ((4k - 1)/3)\sqrt{F} + (7k - 2)^2/108 > 0$. This latter inequality always holds for $k < 2 - \sqrt{3}$, and thus is always satisfied in the case under consideration. Thus, M implements a downstream duopoly resulting in quantities $q_I^u = (2 + 5k)/12$ and $q_E^u = (2 - 7k)/12$. This establishes the desired result. ■

Proposition 1: *(i)* $W^d > W^u$ *if and only if*

> *(1.)* $k < 2 - \sqrt{3}$ *and* $(1/6) - (7/12)k + (\sqrt{1 - 4k - k^2})/12 \leq \sqrt{F} < (1/3) - (8/9)k$, *or*
>
> *(2.)* $2 - \sqrt{3} \leq k \leq 3/10$ *and* $\sqrt{F} < (1/3) - (8/9)k$, *or*
>
> *(3.)* $3/10 < k < 17/46$ *and* $\sqrt{F} < \sqrt{(23/72)k^2 - (5/18)k + (17/288)}$.

(ii) $W^d < W^u$ *if and only if*

(1.) $k < 2-\sqrt{3}$ *and* $\sqrt{F} < (1/6)-(7/12)k+(\sqrt{1-4k-k^2})/12$, *or*

(2.) $\sqrt{F} > (1/3)-(8/9)k$ *for* $k < 3/10$ *or* $\sqrt{F} > \sqrt{(23/72)k^2-(5/18)k+(17/288)}$ *for* $k \geq 3/10$, *and* $\sqrt{F} < (1/3)-(2/3)k$.

(iii) If $\sqrt{F} \geq (1/3)-(2/3)k$, *then* $W^d = W^u$.

Proof:

First, note that for $k \in (0, 2-\sqrt{3}]$, $(1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12 < (1/3)-(2/3)k$, $(1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12 = (1/6)-(1/3)k$ if and only if $k = (\sqrt{3}-1)/4$, and $(1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12 = (1/6)-(7/12)k$ if and only if $k = 2-\sqrt{3}$. These observations together with Lemmas 2 and 3 imply that there are five cases to consider that we labeled with Roman numerals in Figure 6.

(I) $k < 2-\sqrt{3}$ and $\sqrt{F} \leq (1/6)-(7/12)k$:

Under both pricing regimes, M implements an unconstrained downstream duopoly, resulting in the same aggregate output, $Q^d = Q^u = (2-k)/6$. Under price discrimination, however, the less efficient firm E produces a higher share of output, $q_E^d = (1-2k)/6 > (2-7k)/12 = q_E^u$. Thus, welfare is strictly lower under price discrimination than under uniform pricing, $W^d < W^u$.

(II) $k < 2-\sqrt{3}$, $(1/6)-(7/12)k < \sqrt{F} < (1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12$, and $\sqrt{F} \leq (1/6)-(1/3)k$:

Under price discrimination, M implements an unconstrained duopoly resulting in quantities $q_I^d = (1+k)/6$, $q_E^d = (1-2k)/6$, and $Q^d = (2-k)/6$, whereas under uniform pricing, M implements a constrained duopoly, resulting in quantities $q_I^u = k+\sqrt{F}$, $q_E^u = \sqrt{F}$, and $Q^u = k+2\sqrt{F}$. $(1/6)-(7/12)k < \sqrt{F}$ implies that aggregate output is larger under uniform pricing than under price discrimination, $Q^d < Q^u$. $\sqrt{F} \leq (1/6)-(1/3)k$, on the other hand, implies, that the less efficient firm's output is (at least weakly) lower under uniform pricing than under price discrimination. Together, these observations imply that welfare under uniform pricing exceeds welfare under price discrimination, $W^d < W^u$.

(III) $k < (\sqrt{3}-1)/4$ and $(1/6)-(1/3)k < \sqrt{F} < (1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12$: Under both pricing regimes, M implements a constrained duopoly. Under price discrimination, this results in in quantities $q_I^d = (1/4) - (1/2)\sqrt{F}$, $q_E^d = \sqrt{F}$, and $Q^d = (1/4) + (1/2)$. Under uniform pricing, the resulting quantities are $q_I^u = k + \sqrt{F}$, $q_E^u = \sqrt{F}$, and $Q^u = k + 2\sqrt{F}$. While the less efficient firm's output being identical under both pricing regimes, $q_E^d = q_E^u = \sqrt{F}$, $(1/6) - (1/3)k \leq \sqrt{F}$ implies that aggregate output is higher under uniform pricing than under price discrimination, $Q^d < Q^u$. This, in turn, implies that welfare under uniform pricing exceeds welfare under price discrimination, $W^d < W^u$.

(IV) $(1/6)-(1/3)k < \sqrt{F} < (1/3)-(2/3)k$ and $(1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12 \leq \sqrt{F}$:

Under price discrimination, M implements a constrained downstream duopoly, resulting in quantities $q_I^d = (1/4) - (1/2)\sqrt{F}$, $q_E^d = \sqrt{F}$, and $Q^d = (1/4) + (1/2)\sqrt{F}$. Welfare under this pricing regime then is given by

$$W^d = \int_0^{Q^d} (1-x)dx - kq_E^d - F = \frac{7}{32} + \left(\frac{3}{8} - k\right)\sqrt{F} - \frac{9}{8}(\sqrt{F})^2. \quad \text{(C.5)}$$

Under uniform pricing, on the other hand, M implements an unconstrained downstream monopoly with I as the downstream monopoly firm, resulting in quantity $q_I^u = Q^u = 1/4$. Welfare under this pricing regime then is given by

$$W^u = \int_0^{Q^u} (1-x)dx = \frac{7}{32}. \quad \text{(C.6)}$$

With $F > 0$, $W^d > W^u$ if and only if $\sqrt{F} < (1/3) - (8/9)k$. Obviously, for all $k \in (0, 0.5)$ we have $(1/3) - (8/9)k < (1/3) - (2/3)k$. Moreover, for $k \in (0, 2-\sqrt{3}]$, $(1/3) - (8/9)k > (1/6) - (7/12)k + (\sqrt{1-4k+k^2})/12$. Last, note that $(1/3) - (8/9)k$ and $(1/6) - (1/3)k$ intersect at $k = 0.3$. Thus, $W^d > W^u$ if and only if $k < 0.3$ and $(1/6) - (1/3)k < \sqrt{F}$, $(1/6) - (7/12)k + (\sqrt{1-4k+k^2})/12 \leq \sqrt{F}$, and $\sqrt{F} < (1/3) - (8/9)k$.

(V) $k \geq (\sqrt{3}-1)/4$, $\sqrt{F} \leq (1/6)-(1/3)k$, and $\sqrt{F} \geq (1/6)-(7/12)k + (\sqrt{1-4k+k^2})/12$ for $k \in [(\sqrt{3}-1)/4, 2-\sqrt{3})$:

Under price discrimination, M implements an unconstrained duopoly resulting in quantities $q_I^d = (1+k)/6$, $q_E^d = (1-2k)/6$, and $Q^d = (2-k)/6$. Welfare under this pricing regime then is given by

$$W^d = \int_0^{Q^d} (1-x)dx - kq_E^d - F = \frac{20-20k+23k^2}{72} - F. \tag{C.7}$$

Under uniform pricing, on the other hand, M implements an unconstrained downstream monopoly with I as the downstream monopoly firm, resulting in quantity $q_I^u = Q^u = 1/4$. Welfare under this pricing regime then is given by

$$W^u = \int_0^{Q^u} (1-x)dx = \frac{7}{32}. \tag{C.8}$$

Thus, $W^d > W^u$ if and only if $F < (23/72)k^2 - (5/18)k + (17/288) =: F_W(k)$. Note that $F_W(k) > 0$ for $k < 17/46$ and $F_W(k) \leq 0$ for $k \in [17/46, 0.5]$. With $F_W(k) > 0$ for $k < 17/46$, it is readily verified that $d\sqrt{F_W(k)}/dk < 0$ for $k \leq 17/46$. Moreover, $\sqrt{F_W(k)} = (1/6)-(1/3)k$ if and only if $k = 0.3$. Thus, $W^d \leq W^u$ if and only if $k \geq 0.3$ and $\sqrt{F_W(k)} \leq \sqrt{F} \leq (1/6)-(1/3)k$.

Last, for $\sqrt{F} \geq (1/3)-(2/3)k$ M implements a downstream monopoly with I as the downstream monopoly firm under both pricing regimes, resulting in quantity $q_I^d = q_I^u = Q^d = Q^u = 1/4$. Thus, there is no difference in welfare under both pricing regimes, $W^d = W^u$. Combining these observations establishes the desired result. ■

D. Downstream Competition with a More Efficient Entrant

In this appendix, we formally (but only briefly) discuss the case of downstream Cournot competition with a more efficient entrant. We restrict attention to $k \leq 1/4$ and $f \leq (1/2)(1-k)$ with $\sqrt{F} = f$. First, we consider price discrimination and thereafter uniform pricing. Finally, we compare welfare under the two pricing regimes.

Price Discrimination.—Suppose the entry constraint does not impose a binding restriction on the manufacturer's choice of wholesale prices. The optimal wholesale prices are:

$$w_I^d = (1/2)(1-k), \qquad w_E^d = 1/2. \tag{D.1}$$

In this case, the manufacturer's profit and the welfare is given by

$$\begin{aligned} \Pi &= (1/6)(1-k+k^2) && \text{(D.2)} \\ \text{and} \quad W &= (1/72)(20-20k+23k^2) - f^2, && \text{(D.3)} \end{aligned}$$

respectively. The manufacturer is indeed unrestricted if and only if

$$f < (1/6)(1+k) \equiv \bar{f}^d(0). \tag{D.4}$$

For higher values of the entry cost, the entry constraint imposes a binding restriction in the manufacturer's optimization problem. If it is nevertheless optimal to serve both downstream firms, then the equilibrium wholesale prices, manufacturer's profit, and welfare are:

$$\begin{aligned} w_I^D &= (1/2)(1-k) && \text{(D.5)} \\ w_E^D &= (1/4)(3+k-6f) && \text{(D.6)} \\ \Pi &= (1/8)(1+4f-12f^2-2k+4fk+k^2) && \text{(D.7)} \\ W &= (1/32)(7+12f-4f^2-14k+20fk+7k^2) - f^2. && \text{(D.8)} \end{aligned}$$

With the wholesale prices given by (D.5) and (D.6) the incumbent demands a positive quantity only if $f < (1/2)(1-k)$.

The manufacturer has the entrepreneurial freedom to serve only the incumbent. The optimal wholesale price in this case is $w^M = (1/2)(1-k)$, and the manufacturer gains a profit of $\Pi = (1/8)(1-k)^2$. The welfare is $W = (7/32)(1-k)^2$.

Thus, the manufacturer prefers to serve only the incumbent instead of making a special offer to the entrant—that would allow the entrant to break even—if

$$f > (1/3)(1+k) \equiv \hat{f}^d(0). \tag{D.9}$$

It can be shown that under the imposed restrictions on (f, k) it is never optimal to serve only the entrant.

Uniform Pricing.—For low values of the fixed cost, the entry constraint does not impose a binding restriction. If this is the case, the optimal uniform wholesale price, the manufacturer's profit, and welfare are given by

$$\begin{aligned} w^u &= (1/4)(2-k) && \text{(D.10)} \\ \Pi &= (1/24)(4-4k+k^2) && \text{(D.11)} \\ W &= (1/72)(20-20k+41k^2)-f^2. && \text{(D.12)} \end{aligned}$$

Given the above wholesale price, the entrant makes a positive profit if

$$f < (1/6) + (5/12)k \equiv \bar{f}^u(0). \quad \text{(D.13)}$$

Moreover, notice that the incumbent demands a positive quantity if $k < 2/7$, which is always satisfied.

Now, suppose the entry constraint imposes a binding restriction. The uniform wholesale price is determined by the entrant's break-even constraint,

$$w^u = 1 + k - 3f. \quad \text{(D.14)}$$

With this wholesale price, the manufacturer's profit and welfare are

$$\begin{aligned} \Pi &= 2f - 6f^2 - k + 5fk - k^2 && \text{(D.15)} \\ W &= (1/2)(4f - 4f^2 - 2k + 2fk + k^2) - f^2. && \text{(D.16)} \end{aligned}$$

The quantity procured by the incumbent is positive if $f > k$.

As in the price discrimination case, the manufacturer may prefer to serve only the incumbent at wholesale price $w^M = (1/2)(1-k)$ instead of offering both firms the wholesale price that allows the entrant just to break even. It can be shown that it is optimal to serve only the incumbent if

$$f > \frac{1}{12}\left(2 + 5k + \sqrt{1 + 2k - 2k^2}\right) \equiv \hat{f}^U(0). \quad \text{(D.17)}$$

Note that $(1/6) + (1/2)k < \hat{f}^u(0) < (1/3) + (1/3)k$ for $k \leq 1/4$. Moreover, in the range of parameter values under consideration, if it is optimal to serve only the incumbent, then this is also a feasible strategy. Finally, one can show that it is never optimal for the manufacturer to serve only the entrant if $k \leq 1/4$.

Welfare.—Define $\Delta W := W^d - W^u$. We have to distinguish five cases.

Case I: $f < \bar{f}^d(0)$.

There is an unrestricted downstream duopoly under both pricing regimes. Comparing the two welfare expressions immediately reveals that $\Delta W < 0$.

Case II: $\bar{f}^d(0) < f < \bar{f}^u(0)$.

We have a duopoly downstream under both pricing regimes. The entry constraint imposes a binding restriction under price discrimination but not under uniform pricing. The difference in welfare is

$$\Delta W = \frac{1}{288}[-17 - 36f^2 - 46k - 101k^2 + 36f(3 + 5k)]. \tag{D.18}$$

We solve for the f-values such that $\Delta W = 0$. Since ΔW is a quadratic expression in f we obtain two roots. Only one of the two roots is relevant and given by

$$f^W(0) \equiv \frac{1}{6}\left[9 + 15k - 2\sqrt{16 + 56k + 31k^2}\right]. \tag{D.19}$$

Note that $\bar{f}^d(0) < f^W(0) < \bar{f}^u(0)$ for $k \in [0, 1/4]$. In summary, given case II, we have $\Delta W > 0$ if and only if $f > f^W(0)$.

Case III: $\bar{f}^u(0) < f < \hat{f}^u(0)$.

The downstream market structure is a duopoly. Under both pricing regimes the entry constraint imposes a binding restriction on the manufacturer in his optimal choice of wholesale prices. The difference in welfare is given by

$$\Delta W = \frac{1}{32}[7 - 52f + 60f^2 + 18k - 12fk - 9k^2]. \tag{D.20}$$

By solving for the roots of ΔW—the f-values such that $\Delta W = 0$—we obtain two solutions. For f-values between these two roots, we have $\Delta W < 0$. It can be shown that one root is always negative, while the other one always exceeds unity.

Case IV: $\hat{f}^u(0) < f < \hat{f}^d(0)$.

Entry occurs only under price discrimination, while under uniform pricing the incumbent monopolizes the downstream market. Here,

$$\Delta W = \frac{1}{8}f[3 - 9f + 5k]. \tag{D.21}$$

Observe that $\Delta W > 0$ if and only if $f < (1/3) + (5/9)k$. Hence, in case IV. welfare is higher under price discrimination.

Case V: $f \geq \hat{f}^d(0)$.

Only the incumbent is served under either pricing regime. Obviously, here we have $\Delta W = 0$.