Supplementary Material to "Discriminatory Nonlinear Pricing, Fixed Costs, and Welfare in Intermediate-Goods Markets"

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Appendix B. Supplementary Material to "General Nonlinear Tariffs" (Proof of Proposition 4)

Instead of two-part tariffs, the manufacturer now offers menus of quantity-transfer pairs to the downstream firms. Here, downstream firms acquire a given quantity at a pre-specified price, such that they indeed might have an incentive not to convert all units of the input into the final product. In this regard, the assumption of free disposal gives leeway in decision-making to the downstream firms and weakens the position of the manufacturer. In what follows, we initially abstract from free disposal of the input good; i.e., we consider quantity forcing contracts, under which a downstream firm sells the same amount of the final consumption good as it acquired from the input good. In the end, we will show that—under the additional assumption that $K > k$—the allocation implemented by the optimal quantity forcing contract also prevails under free disposal, such that this contract must also be optimal if downstream firms can freely dispose of the input.

Slightly abusing the notation introduced in the paper, let

$$\pi(q, k_i) = q[P(q) - k_i] \tag{B.1}$$

denote the "gross" profits (i.e., profits before subtracting any transfer payment and the fixed cost) of a firm from selling quantity q when operating at marginal cost $k_i \in \{0, k\}$.

Price Discrimination.—If price discrimination is allowed, the manufacturer can offer each retailer an individualized contract and therefore, effectively, solves two independent optimization problems. Consider the manufacture's contract offer to downstream firm $i \in \{0, k\}$ that operates at marginal cost k_i, where $k_0 = 0 < k = k_k$, and associated fixed cost F_i. The logic of the revelation principle implies that maximum upstream profits can be achieved by an individually rational contract of the form $\{(q_i, t_i)\}$. In consequence, the manufacturer solves the following problem:

$$\begin{aligned} &\max_{(q_i,t_i)} \; t_i - Kq_i \\ &\text{subject to} \\ &(\text{PC}_i) \quad \pi(q_i, k_i) - t_i \geq F_i \end{aligned}$$

Under the optimal contract, the (PC_i) constraint has to bind. In consequence, the manufacturer effectively chooses quantity q_i to maximize the profits of a vertically integrated firm, which leads to the following observation:

Lemma 3. *Let q_i^d denote the optimal quantity offered to retailer $i \in \{0, k\}$ under price discrimination. Then, $q_i^d = q^{JS}(k_i)$.*

Uniform Pricing.—If price discrimination is banned, the manufacturer has to offer one and the same contract to both downstream firms. As there are two types of downstream firms, the logic of the revelation principle implies that maximum upstream profits can be achieved by an incentive compatible and individually rational menu of the form $\{(q_0, t_0), (q_k, t_k)\}$, where (q_i, t_i) is the quantity-transfer pair designated to downstream firm $i \in \{0, k\}$. In consequence, the manufacturer solves the following problem:

$$\begin{aligned} &\max_{(q_0,t_0),(q_k,t_k)} \; [t_0 - Kq_0] + [t_k - Kq_k] \\ &\text{subject to} \\ &(\text{PC}_0) \quad \pi(q_0, 0) - t_0 \geq F_0 \\ &(\text{PC}_k) \quad \pi(q_k, k) - t_k \geq F_k \\ &(\text{IC}_0) \quad \pi(q_0, 0) - t_0 \geq \pi(q_k, 0) - t_k \\ &(\text{IC}_k) \quad \pi(q_k, k) - t_k \geq \pi(q_0, k) - t_0 \end{aligned}$$

We start with some basic, yet important, observations. First, both (IC_0) and (IC_k) being simultaneously satisfied implies that the following monotonicity requirement is satisfied:

$$q_k \leq q_0. \tag{MON}$$

Second, under the optimal contract, either (PC_i) or (IC_i) (or both) has to be binding. Third, if the monotonicity requirement is satisfied and one retailer's incentive compatibility constraint binds, then the other retailer's incentive compatibility constraint is automatically satisfied. Hence, under the optimal contract, at most one incentive compatibility constraint imposes a binding restriction.

Which constraints actually impose a binding restriction is determined by the ratio of the difference in fixed costs to the difference in marginal costs, $\frac{F}{k}$ with $F := F_0 - F_k$. We next present a detailed analysis of the cases that have to be distinguished.

Case I: (IC_0) and (PC_k) bind.
If (IC_0) and (PC_k) are binding, transfers are given by

$$t_0 = \pi(q_0, 0) - kq_k - F_k \qquad \text{and} \qquad t_k = \pi(q_k, k) - F_k. \tag{B.2}$$

Ignoring (IC_k) and (PC_0) for the moment, the manufacturer's problem amounts to

$$\max_{q_0, q_k} [\pi(q_0, 0) - Kq_0] + [\pi(q_k, k) - (K + k)q_k] - 2F_k \tag{B.3}$$

From the manufacturer's objective function it becomes apparent that the optimal quantity to offer the retailer with low marginal cost is $q_0^I = q^{JS}(0)$. Differentiation w.r.t. q_k reveals that the optimal quantity to offer the retailer with high marginal cost is $q_k^I = 0$ if $P(0) \leq 2k + K$. If $P(0) > 2k + K$, on the other hand, the retailer with high marginal cost is offered a strictly positive quantity $q_k^I > 0$, which is implicitly characterized by

$$P(q_k^I) + q_k^I P'(q_k^I) = K + 2k. \tag{B.4}$$

It remains to check whether the neglected constraints are satisfied. By assumption the left-hand side of (B.4) is decreasing in q, such that $q_k^I < q^{JS}(k)$. This, in turn, implies that the monotonicity requirement (MON) is satisfied. As (IC_0) binds, (IC_k) then is automatically satisfied. Finally, (PC_0) is satisfied as long as

$$F_0 \leq kq_k^I + F_k \iff q_k^I \geq \frac{F}{k}. \tag{B.5}$$

Case II: (IC_0), (PC_0), and (PC_k) bind.
If (PC_0) and (PC_k) are binding, transfers are given by

$$t_0 = \pi(q_0, 0) - F_0 \qquad \text{and} \qquad t_k = \pi(q_k, k) - F_k. \tag{B.6}$$

Inserting these transfers into the binding (IC_0) constraint pins down the quantity optimally offered to the retailer with high marginal cost:

$$F_0 = \pi(q_k, 0) - \pi(q_k, k) + F_k \;\Rightarrow\; q_k^{II} = \frac{F_0 - F_k}{k}. \tag{B.7}$$

Ignoring (IC_k) for the moment, the manufacturer's problem amounts to

$$\max_{q_0} \, [\pi(q_0, 0) - Kq_0] + [\pi(q_k^{II}, k) - Kq_k^{II}] - F_0 - F_k. \tag{B.8}$$

Hence, the optimal quantity to offer the retailer with low marginal cost is $q_0^{II} = q^{JS}(0)$.

The monotonicity requirement (MON) is satisfied as long as

$$q^{JS}(0) \geq \frac{F}{k}, \tag{B.9}$$

in which case also (IC_k) is satisfied because (IC_0) binds.

Case III: (PC_0) and (PC_k) bind.
If (PC_0) and (PC_k) are binding, transfers are given by

$$t_0 = \pi(q_0, 0) - F_0 \qquad \text{and} \qquad t_k = \pi(q_k, k) - F_k. \tag{B.10}$$

Ignoring (IC_0) and (IC_0) for the moment, the manufacturer's problem amounts to

$$\max_{q_0, q_k} \, [\pi(q_0, 0) - Kq_0] + [\pi(q_k, k) - Kq_k] - F_0 - F_k \tag{B.11}$$

From the manufacturer's objective function it becomes apparent that the optimal quantity to offer the retailer with low marginal cost is $q_0^{III} = q^{JS}(0)$. Likewise, the optimal quantity to offer the retailer with high marginal cost is $q_k^{III} = q^{JS}(k)$.

It remains to check whether the neglected constraints are satisfied. First, (IC_0) is satisfied as long as

$$F_0 \geq kq^{JS}(k) + F_k \iff q^{JS}(k) \leq \frac{F}{k}. \tag{B.12}$$

Likewise, (IC_k) is satisfied as long as

$$F_k \geq -kq^{JS}(0) + F_0 \iff q^{JS}(0) \geq \frac{F}{k}. \tag{B.13}$$

Case IV: (PC_0), (PC_k), and (IC_k) bind.
If (PC_0) and (PC_k) are binding, transfers are given by

$$t_0 = \pi(q_0, 0) - F_0 \qquad \text{and} \qquad t_k = \pi(q_k, k) - F_k. \tag{B.14}$$

Inserting these transfers into the binding (IC_k) constraint pins down the quantity optimally offered to the retailer with low marginal cost:

$$F_k = \pi(q_0, k) - \pi(q_0, 0) + F_0 \;\Rightarrow\; q_0^{IV} = \frac{F}{k}. \tag{B.15}$$

Ignoring (IC_0) for the moment, the manufacturer's problem amounts to

$$\max_{q_k} \, [\pi(q_0^{IV}, 0) - Kq_0^{IV}] + [\pi(q_k, k) - Kq_k] - F_0 - F_k. \tag{B.16}$$

Hence, the optimal quantity to offer the retailer with high marginal cost is $q_0^{IV} = q^{JS}(k)$.

The monotonicity requirement (MON) is satisfied as long as

$$q^{JS}(k) \leq \frac{F}{k}, \tag{B.17}$$

in which case also (IC_0) is satisfied because (IC_k) binds.

Case V: (PC_0) and (IC_k) bind.
If (PC_0) and (IC_k) are binding, transfers are given by

$$t_0 = \pi(q_0, 0) - F_0 \qquad \text{and} \qquad t_k = \pi(q_k, k) + kq_0 - F_0. \tag{B.18}$$

Ignoring (IC_0) and (PC_k) for the moment, the manufacturer's problem amounts to

$$\max_{q_0, q_k} \, [\pi(q_0, 0) - (K - k)q_0] + [\pi(q_k, k) - Kq_k] - 2F_0 \tag{B.19}$$

From the manufacturer's objective function it becomes apparent that the optimal quantity to offer the retailer with high marginal cost is $q_k^V = q^{JS}(k)$. Differentiation w.r.t. q_0 reveals that the optimal quantity to offer the retailer with low marginal cost is $q_0^V > 0$, which is implicitly characterized by

$$P(q_0^V) + q_0^V P'(q_0^V) = K - k. \tag{B.20}$$

It remains to check whether the neglected constraints are satisfied. The left-hand side of (B.20) is decreasing in q, such that $q_0^V > q^{JS}(0)$. This, in turn, implies that the monotonicity requirement (MON) is satisfied. As (IC_k) binds, (IC_0) then is automatically satisfied. Finally, (PC_k) is satisfied as long as

$$F_k \leq -kq_0^V + F_0 \iff q_0^V \leq \frac{F}{k}. \tag{B.21}$$

Noting that the manufacturer will always be (weakly) better off in a situation where only two (Cases I, III and V) rather than three constraints (Cases II and IV) impose a binding restriction, the following result summarizes the optimal quantities under uniform pricing.

Lemma 4. *Let q_i^u denote the optimal quantity offered to downstream firm $i \in \{0, k\}$ under uniform pricing. Then:*

(i) If $\frac{F}{k} \leq q_k^I$, then $q_k^u = q_k^I$ and $q_0^u = q^{JS}(0)$.

(ii) If $q_k^I < \frac{F}{k} < q^{JS}(k)$, then $q_k^u = \frac{F}{k}$ and $q_0^u = q^{JS}(0)$.

(iii) If $q^{JS}(k) \leq \frac{F}{k} \leq q^{JS}(0)$, then $q_k^u = q^{JS}(k)$ and $q_0^u = q^{JS}(0)$.

(iv) If $q^{JS}(0) < \frac{F}{k} < q_0^V$, then $q_k^u = q^{JS}(k)$ and $q_0^u = \frac{F}{k}$.

(v) If $q_0^V \leq \frac{F}{k}$, then $q_k^u = q^{JS}(k)$ and $q_0^u = q_0^V$.

Free Disposal.—Lemmas 3 and 4 characterize the optimal quantities for the case of quantity forcing. Now suppose that downtream firms can freely dispose of the input. Given downstream firm $i \in \{0, k\}$ obtains quantity $\tilde{q}$ of the input for free, it will sell $\min\{\tilde{q}, q(k_i)\}$ units of the final product, where $q(k_i)$ is defined in (1) and satisfies $P(q(k_i))+q(k_i)P'(q(k_i)) = k_i$. With $q^{JS}(k)$ and q_0^V being defined by $P(q^{JS}(k)) + q^{JS}(k)P'(q^{JS}(k)) = K + k > k$ and $P(q_0^V) + q_0^V P'(q_0^V) = K - k > 0$, respectively, and $P(q) + qP'(q)$ bing strictly decreasing, it follows that $q_k^u \leq q_k^d = q^{JS} < q(k)$ and $q_0^d \leq q_0^u \leq q_0^V < q(0)$. In consequence, free disposal leaves the quantities sold by each downstream firm under each pricing regime unaffected. The quantities that the manufacturer offers to the two downstream firms under the respective pricing regime, as characterized in Lemmas 3 and 4, are depicted in Figure B.1

Welfare.—Social welfare does not depend on the specifics of the contractual form, but on the quantities of the final consumption good. As

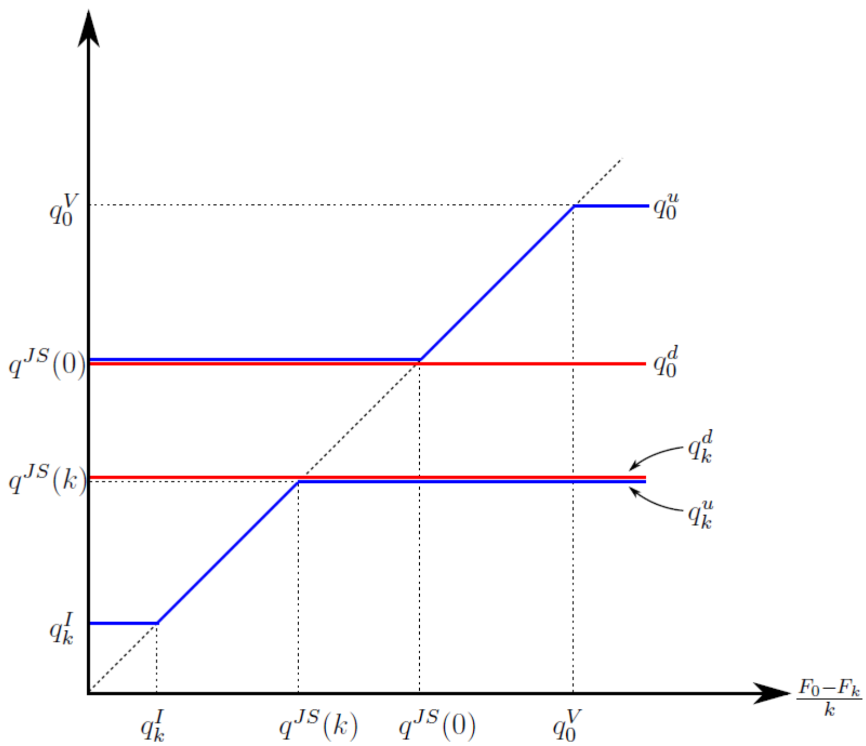

Figure B.1: Optimal quantities under general nonlinear contracts.

$P(0) > K + k$ by assumption, each market should be served from a welfare perspective. The quantity that maximizes welfare in the market served by downstream firm $i \in \{0, k\}$, q_i^W, maximizes $W_i = \int_0^q P(z)dz - (K + k_i)q$ and thus is characterized by

$$P(q_i^W) = K + k_i. \tag{B.22}$$

Regarding the market served by the downstream firm with marginal cost k, as $P(q_k^W) = K + k < K + k - q^{JS}(k)P'(q^{JS}(k)) = P(q^{JS}(k))$ and $P' < 0$ (whenever $P > 0$), we must have $q^{JS}(k) < q_k^W$. Regarding the market served by the downstream firm with marginal cost 0, note that we must have $t_0^V - Kq_0^V \geq 0$, otherwise the manufacturer would be better off in Case V by offering only the quantity-transfer pair (q_k^{JS}, t_k^{JS}), which would be rejected by the downstream firm with marginal cost 0. With $t_0^V = q_0^V P(q_0^V) - F_0$, this implies $q_0^V[P(q_0^V) - K] \geq F_0$. As, in Case V, $F_0 > F_k \geq 0$, we must have $P(q_0^V) > K = P(_0^W)$, such that $q^{JS}(0) < q_k^W$ as $P' < 0$ (whenever $P > 0$). With W_i being strictly concave in q (whenever $P > 0$), welfare in market $i \in \{0, k\}$ is higher under the pricing regime that leads to the larger quantity being sold in that market. The welfare comparison across pricing regimes stated in Proposition 4 then is immediate.