Appendix A. Supplementary Material to “Salience, Competition, and Decoy Goods” by F. Herweg, D. Müller, and P. Weinschenk

Proof of Proposition 2.

Part (i): Suppose \( p^*_b \) is such that \( q_b/p^*_b > q_f/c_f \). Consider a decoy good with price \( p_d = p^*_b \) and quality \( q_d = \alpha q_b + (1 - \alpha)q_f \), with \( \alpha \in (0, 1) \). The reference good is

\[
\bar{q}_c = \frac{1 + \alpha}{3} q_b + \frac{2 - \alpha}{3} q_f, \quad \bar{p}_c = \frac{2}{3} p^*_b + \frac{1}{3} c_f.
\]

We now show that, for suitable levels of \( \alpha \), this decoy good is appropriate. First, note that if constraint (SC\(_b\)) is satisfied such that quality is salient for the brand product, then constraint (DC) is satisfied for all \( \alpha \in (0, 1) \) because the brand product dominates the decoy good.

It thus remains to show that there are levels of \( \alpha \) such that the two salience constraints, (SC\(_b\)) and (SC\(_f\)), are satisfied. Notice that for all \( \alpha \in (0, 1) \) it holds that \( q_b > \bar{q}_c > q_f \) and \( p^*_b > \bar{p}_c > c_f \), where we have used that \( q_b > q_f \) and \( p^*_b > c_b > c_f \). Thus, neither the brand nor the fringe product dominates the reference good or is dominated by it. By Proposition 1 of Bordalo et al. (2013), the salience constraints are equivalent to

\[
\frac{q_b}{p_b^*} > \frac{\bar{q}_c}{\bar{p}_c}, \quad (SC_b)
\]

\[
\frac{q_f}{c_f} < \frac{\bar{q}_c}{\bar{p}_c}, \quad (SC_f)
\]

Inequality (SC\(_b\)) is equivalent to

\[
\alpha < 1 + \frac{q_b c_f - p^*_b q_f}{p^*_b (q_b - q_f)} =: \hat{\alpha}_b.
\]

By assumption it holds that \( q_b/p^*_b > q_f/c_f \) and thus \( \hat{\alpha}_b > 1 \). Hence, constraint (SC\(_b\)) is always satisfied. Inequality (SC\(_f\)) is equivalent to

\[
\alpha > \frac{q_f (p^*_b - c_f) + p^*_b q_f - c_f q_b}{c_f (q_b - q_f)} =: \hat{\alpha}_f.
\]

For \( \alpha \to 1 \) the above inequality simplifies to \( q_b/p^*_b > q_f/c_f \), which holds by assumption. This implies that \( \hat{\alpha}_f < 1 \). Thus, all decoy goods with \( \alpha \in (\hat{\alpha}_f, 1) \) are appropriate.
Finally, note that the constraints are all slack (strict inequalities). Hence, there also exist decoy goods with \( p_d > p_b^* \) and \( q_d \in (q_f, q_b) \).

**Part (ii):** Suppose \( p_b^* \) is such that \( q_b / p_b^* < q_f / c_f \). The brand product’s quality and price are above average in the extended choice set \( \bar{C} \) if and only if \( q_d < 2q_b - q_f \) and \( p_d < 2p_b^* - c_f \). Recall that \( q_b < 2q_b - q_f \) and \( p_b^* < 2p_b^* - c_f \). In this case, according to Proposition 1 of Bordalo et al. (2013), the salience constraint (SC\(_b\)) is satisfied if and only if

\[
\frac{q_b}{p_b} > \frac{\bar{q}_b}{\bar{p}_b} \iff q_d < \frac{q_b}{p_b} p_d + c_f \left( \frac{q_b}{p_b} - \frac{q_f}{c_f} \right) =: \bar{q}(p_d). \tag{A.4}
\]

Likewise, the fringe product’s quality and price are below average in the extended choice set \( \bar{C} \) if and only if \( q_f > 2q_f - q_b \) and \( p_d > 2c_f - p_b^* \). Recall that \( q_f > 2q_f - q_b \) and \( c_f > 2c_f - p_b^* \). In this case, according to Proposition 1 of Bordalo et al. (2013), the salience constraint (SC\(_f\)) is satisfied if and only if

\[
\frac{q_f}{c_f} > \frac{\bar{q}_f}{\bar{p}_f} \iff q_d < \frac{q_f}{c_f} p_d - p_b^* \left( \frac{q_b}{p_b} - \frac{q_f}{c_f} \right) =: \bar{q}(p_d). \tag{A.5}
\]

As

\[
\bar{q}(p_d) < \bar{q}(p_d) \iff (p_b^* + c_f + p_d) \left( \frac{q_b}{p_b} - \frac{q_f}{c_f} \right) < 0, \tag{A.6}
\]

for \( p_d \geq 0 \) we have \( \min\{\bar{q}(p_d), \bar{q}(p_d)\} = \bar{q}(p_d) \).

Defining \( \bar{p}_d \) implicitly by \( \bar{q}(\bar{p}_d) = q_b \), we find that

\[
\bar{p}_d = p_b^* - q_f \left( \frac{c_f}{q_f} - \frac{p_b^*}{q_b} \right) \in (p_b^*, 2p_b^* - c_f). \tag{A.7}
\]

In consequence, any decoy good \((q_d, p_d)\) with \( q_d = q_b \) and \( p_d \in (\bar{p}_b, 2p_b^* - c_f) \) is appropriate, as it satisfies not only (SC\(_b\)) and (SC\(_f\)), but also is perceived as strictly inferior to the brand product by every consumer type \( \theta \in [\bar{\theta}, \bar{\theta}] \) irrespective of whether quality or price is salient for the decoy good, i.e., it also satisfies (DC).

Finally, note that

\[
\frac{q_d}{p_d} < \frac{\bar{q}_d}{\bar{p}_d} \iff q_d < \frac{q_b + q_f}{p_b^* + c_f} p_d =: \bar{q}(p_d). \tag{A.8}
\]
As $\bar{q}(p_d) > \hat{q}(p_d)$, for a decoy good with above-average price $p_d \in (\bar{p}_d, 2p_b^* - c_f)$ and above-average quality $q_d \in (q_b, \min\{2q_b - q_f, \hat{q}(p_b)\})$, price is salient according to Proposition 1 of Bordalo et al. (2013). In consequence, as long as $q_d$ is sufficiently close to $q_b$, the price-salient decoy good will still be perceived as strictly inferior to the quality-salient brand product – i.e., there also exist decoy goods with $p_d > p_b^*$ and $q_d > q_b$ that are appropriate.

\textbf{References}