## Appendix A. Supplementary Material to "Salience, Competition, and Decoy Goods" by F. Herweg, D. Müller, and P. Weinschenk

## Proof of Proposition 2.

**Part (i)**: Suppose  $p_b^*$  is such that  $q_b/p_b^* > q_f/c_f$ . Consider a decoy good with price  $p_d = p_b^*$  and quality  $q_d = \alpha q_b + (1 - \alpha)q_f$ , with  $\alpha \in (0, 1)$ . The reference good is

$$\bar{q}_{\hat{\mathcal{C}}} = \frac{1+\alpha}{3}q_b + \frac{2-\alpha}{3}q_f, \ \bar{p}_{\hat{\mathcal{C}}} = \frac{2}{3}p_b^* + \frac{1}{3}c_f.$$
(A.1)

We now show that, for suitable levels of  $\alpha$ , this decoy good is appropriate. First, note that if constraint (SC<sub>b</sub>) is satisfied such that quality is salient for the brand product, then constraint (DC) is satisfied for all  $\alpha \in (0, 1)$  because the brand product dominates the decoy good.

It thus remains to show that there are levels of  $\alpha$  such that the two salience constraints, (SC<sub>b</sub>) and (SC<sub>f</sub>), are satisfied. Notice that for all  $\alpha \in (0, 1)$  it holds that  $q_b > \bar{q}_{\hat{c}} > q_f$  and  $p_b^* > \bar{p}_{\hat{c}} > c_f$ , where we have used that  $q_b > q_f$ and  $p_b^* \ge c_b > c_f$ . Thus, neither the brand nor the fringe product dominates the reference good or is dominated by it. By Proposition 1 of Bordalo et al. (2013), the salience constraints are equivalent to

$$\frac{q_b}{p_b^*} > \frac{\bar{q}_{\hat{\mathcal{C}}}}{\bar{p}_{\hat{\mathcal{C}}}} \tag{SC}_b$$

$$\frac{q_f}{c_f} < \frac{\bar{q}_{\hat{\mathcal{C}}}}{\bar{p}_{\hat{\mathcal{C}}}}.$$
(SC<sub>f</sub>)

Inequality  $(SC_b)$  is equivalent to

$$\alpha < 1 + \frac{q_b c_f - p_b^* q_f}{p_b^* (q_b - q_f)} =: \hat{\alpha}_b.$$
(A.2)

By assumption it holds that  $q_b/p_b^* > q_f/c_f$  and thus  $\hat{\alpha}_b > 1$ . Hence, constraint  $(SC_b)$  is always satisfied. Inequality  $(SC_f)$  is equivalent to

$$\alpha > \frac{q_f(p_b^* - c_f) + p_b^* q_f - c_f q_b}{c_f(q_b - q_f)} =: \hat{\alpha}_f.$$
(A.3)

For  $\alpha \to 1$  the above inequality simplifies to  $q_b/p_b^* > q_f/c_f$ , which holds by assumption. This implies that  $\hat{\alpha}_f < 1$ . Thus, all decoy goods with  $\alpha \in (\hat{\alpha}_f, 1)$  are appropriate.

Finally, note that the constraints are all slack (strict inequalities). Hence, there also exist decoy goods with  $p_d > p_b^*$  and  $q_d \in (q_f, q_b)$ .

**Part (ii)**: Suppose  $p_b^*$  is such that  $q_b/p_b^* < q_f/c_f$ . The brand product's quality and price are above average in the extended choice set  $\hat{\mathcal{C}}$  if and only if  $q_d < 2q_b - q_f$  and  $p_d < 2p_b^* - c_f$ . Recall that  $q_b < 2q_b - q_f$  and  $p_b^* < 2p_b^* - c_f$ ! In this case, according to Proposition 1 of Bordalo et al. (2013), the salience constraint (SC<sub>b</sub>) is satisfied if and only if

$$\frac{q_b}{p_b^*} > \frac{\bar{q}_{\hat{\mathcal{C}}}}{\bar{p}_{\hat{\mathcal{C}}}} \iff q_d < \frac{q_b}{p_b^*} p_d + c_f \left(\frac{q_b}{p_b^*} - \frac{q_f}{c_f}\right) =: \hat{q}(p_d).$$
(A.4)

Likewise, the fringe product's quality and price are below average in the extended choice set  $\hat{C}$  if and only if  $q_d > 2q_f - q_b$  and  $p_d > 2c_f - p_b^*$ . Recall that  $q_f > 2q_f - q_b$  and  $c_f > 2c_f - p_b^*$ ! In this case, according to Proposition 1 of Bordalo et al. (2013), the salience constraint (SC<sub>f</sub>) is satisfied if and only if

$$\frac{q_f}{c_f} > \frac{\bar{q}_{\hat{\mathcal{C}}}}{\bar{p}_{\hat{\mathcal{C}}}} \iff q_d < \frac{q_f}{c_f} p_d - p_b^* \left(\frac{q_b}{p_b^*} - \frac{q_f}{c_f}\right) =: \tilde{q}(p_d).$$
(A.5)

As

$$\hat{q}(p_d) < \tilde{q}(p_d) \iff (p_b^* + c_f + p_d) \left(\frac{q_b}{p_b^*} - \frac{q_f}{c_f}\right) < 0, \tag{A.6}$$

for  $p_d \ge 0$  we have  $\min\{\hat{q}(p_d), \tilde{q}(p_d)\} = \hat{q}(p_d)$ .

Defining  $\bar{p}_d$  implicitly by  $\hat{q}(\bar{p}_d) = q_b$ , we find that

$$\bar{p}_d = p_b^* - q_f \left(\frac{c_f}{q_f} - \frac{p_b^*}{q_b}\right) \in (p_b^*, 2p_b^* - c_f).$$
(A.7)

In consequence, any decoy good  $(q_d, p_d)$  with  $q_d = q_b$  and  $p_d \in (\bar{p}_b, 2p_b^* - c_f)$  is appropriate, as it satisfies not only  $(SC_b)$  and  $(SC_f)$ , but also is perceived as strictly inferior to the brand product by every consumer type  $\theta \in [\underline{\theta}, \overline{\theta}]$  irrespective of whether quality or price is salient for the decoy good, i.e., it also satisfies (DC).

Finally, note that

$$\frac{q_d}{p_d} < \frac{\bar{q}_{\hat{\mathcal{C}}}}{\bar{p}_{\hat{\mathcal{C}}}} \iff q_d < \frac{q_b + q_f}{p_b^* + c_f} p_d =: \bar{q}(p_d).$$
(A.8)

As  $\bar{q}(p_d) > \hat{q}(p_d)$ , for a decoy good with above-average price  $p_d \in (\bar{p}_d, 2p_b^* - c_f)$ and above-average quality  $q_d \in (q_b, \min\{2q_b - q_f, \hat{q}(p_b)\})$ , price is salient according to Proposition 1 of Bordalo et al. (2013). In consequence, as long as  $q_d$  is sufficiently close to  $q_b$ , the price-salient decoy good will still be perceived as strictly inferior to the quality-salient brand product – i.e., there also exist decoy goods with  $p_d > p_b^*$  and  $q_d > q_b$  that are appropriate.

## References

Bordalo, P., Gennaioli, N., Shleifer, A., 2013. Salience and consumer choice. Journal of Political Economy 121 (5), 803–843.