OVERLAPPING EFFORTS IN THE EU EMISSION TRADING SYSTEM

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ABSTRACT. According to the Phase IV (2021-2030) rules of the EU ETS, the total amount of emissions permits allocated to firms is not fixed but endogenous. This implies that a national climate policy that overlaps with the emission trading system can have an impact on total aggregate emissions. Roughly speaking, if firms increase their holdings of emission permits, the total amount of emissions allocated is reduced. This paper investigates analytically how an overlapping national policy affects the decision of an individual firm and the whole industry to bank emission permits. If marginal abatement costs are not too convex, national climate policies increase banking and thus tend to reduce overall emissions. This effect, however, is reduced in times of low interest rates.

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1. INTRODUCTION

The European Union's greenhouse gas emission trading system (EU ETS) is the largest such market in the world.[1] It accounts for 45% of the EU's total greenhouse gas emissions. Recently, the European Parliament and the Council agreed on various changes for the EU ETS, in particular for Phase IV (2021 - 2030). First, in 2019 the so-called *Market Stability Reserve* (MSR) was implemented.[2] Roughly speaking, if in a certain year the *Total Number of Allowances in Circulation* (TNAC) exceeds the threshold of 833 million allowances, then – for this year – the auction volume of allowances is reduced. The difference between the planed and actual auction volume is placed in the MSR. If, on the other hand, the TNAC falls short of 400 million, then additional allowances from the MSR are added to the auction volume of this year.

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[1]That tradable emission permits can implement a given quantity in a cost-effective manner is known since Montgomery (1972). Other notable contributions in this regard are Tietenberg (1985) and Salant (2016). The pros and cons of quantity-based compared to price-based mechanisms are analyzed by Weitzman (1974). A first analysis of an effective mixture of instruments is provided by Roberts and Spence (1976).

[2]See, Decision (EU) 2015/1814.

A second major change will come into force in 2023: The introduction of a *Cancellation Mechanism* (CM).[3] The MSR will obtain a capacity limit, which is given by the auction volume of the previous year. If the rules prescribe that more allowances should be placed in the MSR than it is allowed to store, then these additional allowances are canceled.[4] Due to this change the cap – i.e., total long-run emissions – are not fixed but endogenous. While in a classic cap-and-trade system national measures and individual efforts that overlap with the cap-and-trade system (affect industries regulated by the cap-and-trade system) have no effect on total emissions, this will no longer be true for the EU ETS.

This paper investigates the effect of national measure and individual efforts that overlap with the EU ETS on its outcomes. The national measure could, for instance, be the German shut-down of coal fired power plants. It could also be an effort by individual institutions, like a commitment by a group of universities not to travel by plane for short (within EU) distances. Importantly, I analyze permanent measures and not temporarily ones. I build a model of an intertemporal market for trading emission permits, following Rubin (1996). In particular, I derive comparative statics with respect to the number (the mass) of firms active in the market. The assumption is that the national measure permanently reduces the number of active plants, e.g., shut-down of German coal-fired power plants. The main variable of interest is the TNAC, the aggregate amount of emission permits hold by the firms – also called the aggregate bank of permissions.[5] While not modeling the CM explicitly, I presume that more allowances are canceled, the higher the aggregate bank is. First, and not surprisingly, a national measure that reduces the number of firms always leads to a lower permit price path. This lower price path translates into a higher individual and a higher aggregate bank if the marginal abatement costs are not too convex. Thereby, I show that the typical assumption of quadratic abatement costs is a sufficient condition for this effect. In these cases, national climate policies that overlap with the EU ETS are likely to lead to a reduction of long-run aggregate emissions. I also investigate the role of the interest rate for the outcomes of the EU ETS. In particular, the effect of the currently very low (and even negative) interest rates in the Euro zone on the EU ETS are analyzed. If the interest rate is low, the aggregate bank of permits is large and thus it is more likely that allowances will be

[3]See, Directive (EU) 2018/410.

[4]The cancellation of emission permits will not happen automatically. The final decision is made after an audit of the European Commission and the member states (European Parliament and the Council, 2018).

[5]To be precise, the European Commission (2017) defines the TNAC in year t as follows: $TNAC_t = Supply_t - Demand_t - MSR_t$, where the supply includes banked permits. Thus, a higher aggregate bank of permits translates into a one-to-one increase of the TNAC in that year.

canceled. A low interest rate, however, reduces the effect of overlapping national measures. Moreover, in times of low interest rates, it is unlikely that the MSR will achieve its goal of reducing the actual high surplus of allowances.

The effectiveness of the MSR and other backloading mechanisms within a dynamic cap-and-trade system is analyzed, among others, by Fell (2016); Kollenberg and Taschini (2016); Perino and Willner (2016); Kollenberg and Taschini (2019). Fell (2016) shows that adaptive-allocation mechanisms such as the MSR can reduce overallocation of permits and the volatility of the permit price. He also shows that the effectiveness of these mechanisms is greatly affected by the interest rate. Perino and Willner (2016) and Kollenberg and Taschini (2019), in contrast, find that the MSR tends to increase price volatility.[6] Furthermore, Perino and Willner (2016) point out that the long-run aggregate emissions are neither affected by the MSR nor by overlapping policies. The overlapping policy is modeled as a temporary demand shock affecting the business-as-usual emission of the representative firm. The analysis by Kollenberg and Taschini (2019) has an emphasize on the implications of risk aversion by firms in an uncertain market environment. The optimal adjustment rate of an adaptive-allocation mechanism, taking firms' risk preferences into account, is analyzed by Kollenberg and Taschini (2016).

While the aforementioned papers model adaptive-allocation mechanism, they do not explicitly model a cancellation mechanism. The EU ETS with MSR and CM is analyzed by Carlén, Dahlqvist, Mandell, and Marklund (2019); Bocklet, Hintermayer, Schmidt, and Wildgrube (2019); Beck and Kruse-Andersen (2018). According to the results of Bocklet, Hintermayer, Schmidt, and Wildgrube (2019), the main driver of the expected reduction in aggregate emissions in Phase IV of the EU ETS is the increased linear reduction factor of the annual auction volume, not the CM.[7] The CM also reduces the overall emissions but its main effect is to increase the long-run permit price. One of the first studies that investigates the effects of national measures that overlap with the EU ETS in Phase IV – consisting of MSR and CM – is Carlén, Dahlqvist, Mandell, and Marklund (2019). In a numerical simulation they forecast the effect of a national measure that temporarily reduces emissions by 1 million tons. According to their simulations, the effect on total emissions can be larger than one-to-one. Overall they find that if the measure is implemented early, the effect on total emissions is large (often close to one-to-one). Measures that are

[6] That the MSR can have unintended price effects is also shown by Chaton, Creti, and Sanin (2018).

[7] In Phase III (2013-2020), the allocation of permits decreased annually by 1.74% applied to the average total amount of permits issued annually in Phase II (2008-2012). In Phase IV (2021-2030) the linear reduction factor is 2.2%.

implemented after 2025 are likely to have no or only a small effect on overall emissions. Similar findings are obtained by Beck and Kruse-Andersen (2018): The earlier national policies are implemented, the larger is the effect on total emissions.

While Carlén, Dahlqvist, Mandell, and Marklund (2019); Bocklet, Hintermayer, Schmidt, and Wildgrube (2019) and Beck and Kruse-Andersen (2018) apply numerical methods and presume a linear marginal abatement cost, I derive analytical results without imposing assumptions on marginal abatement costs. Thereby, I show that overlapping national climate policies do not always lead to an increase of the aggregate permission bank. The assumption of a linear marginal abatement cost function, however, is a sufficient condition to guarantee that the aggregate bank increases due to a national policy that overlaps with the cap-and-trade system.

The paper is structured as follows. Section 2 contains the main analysis and the main findings. Subsection 2.1 defines the dynamic market for emission permits. The problem of a representative firm is solved in Subsection 2.2. Comparative statics – overlapping national measures – are analyzed in Subsection 2.3. Further results are derived for specific functional forms in Subsection 2.4. The final Section 3 concludes. All proofs are relegate to the Appendix A.1.

2. Allowance Market and Firm Behavior

2.1. **The Market for Emission Permits.**

In my modeling efforts of the market for emission permits, I closely follow Perino and Willner (2016). There is a continuum of polluting firms with mass $M > 0$. The firms take part in an emission trading system over a fixed time horizon, where time is continuous: A point in time is $t \in [0, T]$ with $T > 0$.[8] A firm is described by its abatement cost function $c(\Delta)$, with $\Delta = u - y(t) \geq 0$. Here, $u > 0$ denotes the business-as-usual (BAU) emissions and $y(t) \geq 0$ denotes the emissions of the firm at time t. The cost function is strictly increasing and strictly convex and satisfies the Inada condition: $c'(\Delta) > 0$, $c''(\Delta) > 0$, and $c'(0) = 0$. At each point in time t, a firm decides on how much to pollute, $y(t) \geq 0$, and how many emission permits to purchase, $x(t)$. Note, $x(t)$ denotes net purchases of permits and thus can be positive as well as negative. A firm can store unused permits for later use, i.e., a firm can bank permits. The individual bank of a firm is denoted by $b(t) \geq 0$. Importantly, a firm can bank permits but is not allowed to borrow (to sell short) permits. For simplicity I assume that $b(0) = 0$.[9] The market interest rate (the discount rate) is denoted by r.

[8]Existing studies point out that overlapping measures have a sizable impact only if implemented early. Therefore, I focus on a finite time horizon. Models with finite time horizon are also analyzed by Fell (2016) and Kollenberg and Taschini (2016).

[9]A positive – and even heterogeneous – initial bank of permits does not change the qualitative findings; see Perino and Willner (2016).

The market for emission allowances is perfectly competitive, i.e., each individual firm takes the price path of allowances as given. The price for one emission permit at time t is $w(t)$. The number of additional allowances allocated by the regulator – typically via an auction – is $S(t)$. I assume that the number of auctioned allowances declines over time, $\dot{S}(t) < 0$. The market for emission permits clears at time t if aggregate net demand equals supply: Formally,

$$Mx(t) = S(t). \tag{1}$$

I assume that emission permits are, in the long-run, a scarce resource:

Assumption 1 (Scarcity of Emission Permits)**.**

$$Mu > \frac{1}{T}\int_{t=0}^{T} S(t)dt. \tag{2}$$

According to Assumption 1, the aggregate BAU emissions of all firms exceed the aggregate number of emission permits.

Finally, I introduce the following notation. Let $h(\cdot)$ be the inverse of the marginal abatement cost function: $h(c'(\Delta)) = \Delta$. By the rules regarding the derivatives of inverse functions, it holds that (i) $h'(\cdot) > 0$ and (ii) $h''(\cdot) \geq 0 \iff c'''(\cdot) \leq 0$.[10]

The purpose of this paper is to investigate the effects on aggregate emissions of permanent national efforts and measures that overlap with the EU ETS in phase IV. The important feature of the EU ETS in phase IV is that the emission cap is endogenous. If the TNAC – the aggregate bank of emission permits – increases, more allowances are placed in the MSR. If the upper limit of the MSR is reached, the additional allowances are canceled. Somewhat simplified, the higher the aggregate bank the lower are total long-run emission. The analysis of the paper is based on this simplified view and does not model the MSR and the CM explicitly. In the following, I investigate the effects of overlapping efforts on the aggregate bank. An overlapping effort – like the German shut-down of coal-fired power plants – permanently reduces the number of firms/plants (M) that are active in the market for emission permits.[11]

[10]Let $h(\cdot) \equiv c'^{-1}(\cdot)$. The first and second derivative is given by (see, e.g. Theorem 4.6 in De la Fuente (2000))

$$h'(\cdot) = \frac{1}{c''(\cdot)} \quad \text{and} \quad h''(\cdot) = -c'''(\cdot)\left(\frac{1}{c''(\cdot)}\right)^3,$$

respectively.

[11]An alternative way to model overlapping efforts is to assume that these efforts reduce the BAU emissions, u; see, e.g. Perino and Willner (2016). If the overlapping measure reduces u, then it leads to an efficient reduction of the emissions (without further regulation). A change in M, on the other hand, corresponds to a situation where the overlapping measure "randomly" shuts down certain plants, i.e., not necessarily the ones with the lowest abatement costs are shut down (or replaced by climate-neutral plants).

2.2. **Cost Minimization Problem & Solution.** Each firm minimizes its aggregated discounted abatement costs:

$$\min_{y(t),x(t)} \int_{t=0}^{T} \{c(u-y(t)) + w(t)x(t)\}\, \mathrm{e}^{-rt} dt \tag{3}$$

subject to

$$\dot{b}(t) = x(t) - y(t), \quad b(0) = 0,\ b(T) = 0 \tag{B}$$

$$b(t) \geq 0 \tag{C}$$

The bank of a firm evolves according to (B): The change in the bank is equal to the net purchases of permits in t minus the emissions in t. The bank is not allowed to be negative and thus a firm has to satisfy constraint (C). Permits are costly and thus it is not optimal to have a positive bank at the final period T, i.e., $b(T) = 0$.

The Hamiltonian associated with the corresponding maximization problem is

$$\mathcal{H}(y, x, p, t) = -\{c(u-y) + w(t)x\}\, \mathrm{e}^{-rt} + p(t)[x - y]. \tag{4}$$

The Lagrangian associated with a firm's problem is

$$\mathcal{L}(y, x, p, \lambda, t) = \mathcal{H}(y, x, p, t) + \lambda(t)b(t), \tag{5}$$

where $\lambda(t) \geq 0$ denotes the Lagrangian multiplier associated with constraint (C); i.e., the bank has to be non-negative.

The necessary conditions for optimality are[12]

$$\mathrm{e}^{-rt}c'(u - y(t)) - p(t) = 0 \tag{6}$$

$$-\mathrm{e}^{-rt}w(t) + p(t) = 0 \tag{7}$$

$$\dot{p}(t) = -\lambda(t) \tag{8}$$

$$\lambda(t) \geq 0 \quad (\lambda(t) = 0 \text{ if } b(t) > 0) \tag{9}$$

I focus on situations where the bank is positive over the whole time span, $b(t) > 0$ for $t \in (0, T)$. This is the case if the interest rate is sufficiently low compared to the reduction of the auctioned volume over time. A sufficient condition is provided in the Appendix A.2. With this assumption, an overlapping measure that changes M, affects the aggregate bank but not the time span for which the bank is strictly positive. In other words, I focus on the direct effect of an overlapping measure on the aggregate bank and ignore the secondary effect that the measure may trigger an extended or reduced banking phase.

If constraint (C) is slack for all $t \in (0, T)$, then $\dot{p}(t) = -\lambda(t) = 0$. This implies that $p(t) = w_0$. From equation (7) it follows that $w(t)\mathrm{e}^{-rt} = w_0$. Next, from

[12]Sufficiency is proven in the Appendix A.2.

differentiating this equality with respect to t, I obtain the Hotelling (1931) rule:

$$\frac{\dot{w}(t)}{w(t)} = r. \tag{10}$$

The price thus evolves according to

$$w(t) = w_o \mathrm{e}^{rt}, \tag{11}$$

where the initial price w_0 – which will be determined endogenously – is a constant shift parameter. From (6) and (7), I obtain that

$$c'(u - y(t)) = w_o \mathrm{e}^{rt}. \tag{12}$$

The above equation determines the optimal intertemporal emission path up to the constant shift parameter w_0. Applying the inverse function of the marginal abatement cost function allows me to write the time t emission as

$$y(t) = u - h(w_0 \mathrm{e}^{rt}). \tag{13}$$

The bank of permissions hold by an individual firm at time t is given by

$$\begin{aligned} b(t) &= \int_0^t [x(z) - y(z)] dz \\ &= \int_0^t \left\{ \frac{1}{M} S(z) - u + h(w_0 \mathrm{e}^{rz}) \right\} dz. \end{aligned} \tag{14}$$

Optimality requires that the bank is depleted at time T: $b(T) = 0$. This condition implicitly characterizes $w_0 = w_0(M)$:

$$\int_0^T \left\{ \frac{1}{M} S(z) - u + h(w_0(M) \mathrm{e}^{rz}) \right\} dz = 0. \tag{15}$$

By Assumption 1 emission permits are a scarce resource and thus – by the above equality – $w_0(M) > 0$.

2.3. **Overlapping Measures.** In this section, I investigate how overlapping measures – e.g. a national policy that substitutes coal-fired power plants by a wind park – affect the market outcomes and firm behavior. In other words, I derive comparative statics with respect to M (the mass of participating firms in the market for emission permits).

Implicit differentiation of (15) with respect to M yields

$$\frac{dw_0}{dM} = \frac{\int_0^T S(t) dt}{M^2 \int_o^T h'(w_0 \mathrm{e}^{rt}) \mathrm{e}^{rt} dt} > 0. \tag{16}$$

The more firms participate in the market for emission permits, the higher is the price path. More interesting than the effect of changing the number of participating firms on the price level is its effect on the banking decision of individual firms. From (14), I obtain that

$$\frac{db(t)}{dM} = \int_0^t \left(-\frac{1}{M^2} S(z) + h'(w_0 \mathrm{e}^{rz}) \mathrm{e}^{rz} \frac{dw_0}{dM} \right) dz. \tag{17}$$

Without further assumptions, the effect of increasing the number of firms M on the bank of an individual firm is undetermined. Under reasonable assumptions, as the next finding shows, the effect is negative.

Proposition 1. *Let $r \geq 0$ and $h'(\alpha) + \alpha h''(\alpha) \geq 0$ for all $\alpha > 0$. Then, at any time $t \in (0, T)$, the individual bank of a firm is higher if the mass of participating firms is lower, i.e,*

$$\frac{db(t)}{dM} < 0 \ \forall \ t \in (0, T). \tag{18}$$

A sufficient condition for $h'(\alpha) + \alpha h''(\alpha) \geq 0$ is that $h''(\cdot) \geq 0$, which is equivalent to $c'''(\cdot) \leq 0$. Thus, if the interest rate is non-negative and if marginal abatement costs are not too convex, then the bank of permits of an individual firm negatively reacts to an expansion of the number of firms.

That the individual bank increases if the number of participating firms is reduced does not imply that the aggregate bank of all firms increases as well. Let $B(t) = Mb(t)$ be the amount of emission permits that is hold at time t by all firms participating in the market for emission permits. The change of the aggregate bank caused by a marginal change in the mass of participating firms is

$$\frac{dB(t)}{dM} = b(t) + M \frac{db(t)}{dM}. \tag{19}$$

The first term is positive while the second term is – most likely – negative. The aggregate bank can be written as follows

$$B(t) = \int_0^t \left[S(z) - Mu + Mh(w_0(M) \mathrm{e}^{rz} \right] dz. \tag{20}$$

The next result shows that the effect of changing the number of firms on the aggregate bank not only depends on the curvature of the marginal abatement cost function but also on the interest rate.

Proposition 2. *Let $h'(\alpha) + \alpha h''(\alpha) \geq 0$ for all $\alpha > 0$. Then,*

(i) *for $r > 0$, the aggregate bank of permits decreases in the number of firms, $dB(t)/dM < 0 \ \forall \ t \in (0, T)$.*
(ii) *for $r < 0$, the aggregate bank of permits increases in the number of firms, $dB(t)/dM > 0 \ \forall \ t \in (0, T)$.*
(iii) *for $r = 0$, the aggregate bank of permits is independent of the number of firms, $dB(t)/dM = 0 \ \forall \ t \in (0, T)$.*

The conditions of Proposition 2 are satisfied if the marginal abatement cost function is not too convex. In this case, if the interest rate is positive, a national measure that reduces M increases the aggregate bank. This effect, however, is reversed if the interest rate is negative. The proposition alludes to the conjecture that the effect of national measures is low in times of low interest rates.[13] While not to be obtained in general, I will illustrate this for an example with specific functional forms.

Before turning to the example, what are the implications of Proposition 2 for overlapping measures within phase IV of the EU ETS? If the aggregate bank increases due to national measures, more permits are placed in the MSR and thus it is also more likely that permits are canceled. Proposition 2(i) shows that – for $r > 0$ – overlapping national measures are likely to have an impact on aggregate emissions. The proposition, however, also points out that this effect vanishes if the interest rate falls to zero.

2.4. **Example: Quadratic Abatement Cost and Linear Reduction Factor.** Most contributions that calibrate the impact of the EU ETS rely on a quadratic abatement cost function. Therefore, I assume that $c(u - y(t)) = \frac{c}{2}(u - y(t))^2$ with $c > 0$. Moreover, the EU ETS – without MSR and CM – has a linear reduction factor, i.e., $S(t) = S_0 - at$ with $0 < a < S_0/T$. For this application, I focus on positive interest rates, $r > 0$.

The emissions of a firm evolve according to

$$y(t) = u - \frac{w_0}{c}\mathrm{e}^{rt}. \tag{21}$$

The price path is, as before, given by the Hotelling rule: $w(t) = w_0\mathrm{e}^{rt}$, where

$$w_0(M, r) = \frac{rc}{\mathrm{e}^{rT} - 1}\left[uT + \frac{aT^2}{2M} - \frac{S_0T}{M}\right] > 0. \tag{22}$$

The initial price w_0 is increasing in the number of firms, M, and decreasing in the interest rate; formally,[14]

$$\frac{\partial w_0}{\partial M} = \frac{rc}{\mathrm{e}^{rT} - 1}\left[\frac{S_0T}{M^2} - \frac{aT^2}{2M^2}\right] > 0, \tag{23}$$

$$\frac{\partial w_0}{\partial r} = \frac{\mathrm{e}^{rT} - 1 - rT\mathrm{e}^{rT}}{(\mathrm{e}^{rT} - 1)^2}\left[uT + \frac{aT^2}{2M} - \frac{S_0T}{M}\right]c < 0. \tag{24}$$

[13]The interest rates applied in numerical analyzes of the EU ETS varies widely, from $r = 3\%$ in Kollenberg and Taschini (2016) to $r = 10\%$ in Perino and Willner (2016). Carlén, Dahlqvist, Mandell, and Marklund (2019) assume that $r = 3.5\%$ and argue that this is consistent with the spread in spot and future prices for 2025 (observed in 2018). Bocklet, Hintermayer, Schmidt, and Wildgrube (2019), on the other hand, assume that $r = 8\%$ – consistent with the approximated weighted average cost of capital of fossil power plants obtained by Kost, Shammugam, Jülch, Nguyen, and Schlegl (2018).

[14]Note that $a \leq S_0/T$ and thus $\partial w_0/\partial M > 0$. In order to see that $\partial w_0/\partial r < 0$ note that $\psi(rT) = -1 - \mathrm{e}^{rT}(rT - 1)$ is strictly decreasing in $rT \geq 0$ and $\psi(0) = 0$.

The aggregate amount of emission permits that is banked by the firms is

$$B(t) = S_0 t - \frac{a}{2}t^2 - Mut + M\frac{e^{rt}-1}{e^{rT}-1}\left[uT + \frac{aT^2}{2M} - \frac{S_0 T}{M}\right]. \tag{25}$$

The aggregate bank of allowances is decreasing in the number of firms

$$\frac{dB(t)}{dM} = -u\left[t - \frac{e^{rt}-1}{e^{rT}-1}\,T\right] < 0. \tag{26}$$

Proposition 3. *Let the abatement cost function be quadratic and suppose that the amount of allowances auctioned declines linearly. Then,*

(i) *the aggregate bank of allowances is decreasing in the interest rate;* $\frac{dB}{dr} < 0$.

(ii) *the negative effect of increasing the number of firms on the aggregate bank is stronger for higher interest rates;* $\frac{d^2B}{drdM} < 0$.

According to Proposition 3, the amount of emission permits held by the firms is high in times of low interest rates. This makes intuitively sense. If the interest rate is low, the optimal time path of emissions declines only gradually. The amount of new emissions allocated thus declines much faster and a firm needs a sizable positive bank in order to be able to implement the optimal intertemporal emission path. These findings are in line with stylized facts of the EU ETS for the time period 2011 till 2017. The interest rates were low and the price for emission permits was almost constant (at a low level). At the same time, firms increased the bank of emission permits significantly.

What are the implications regarding the recent changes of the EU ETS, in particular the introduction of the cancellation mechanism? The currently very low interest rates in the Euro zone will lead to a sustained high aggregate bank of emission permits. Thus, more and more permits will be placed in the MSR and it is very likely that the cap of the MSR is binding for a relatively long time. This would have the implication that many permits will be canceled in the coming years.

As Proposition 3(ii) shows, the effect of overlapping measures – a permanent reduction of the number of firms – is low in times of low interest rates. This suggest the conclusion that recently planned national measures that overlap with the EU ETS will have only a negligible impact on aggregated emissions. In times of low interest rates, however, it is likely that the cap of the MSR is binding. If this is the case, an increase in the size of the bank caused by a national measure may directly translate into an almost one-to-one increase of cancellations. In times of high interest rates, a national measure has a strong impact on the aggregate bank. This increase in the bank, however, may not lead to a significant increase in cancellations because in such times it is unlikely that the cap is binding. A precise analysis of these effect requires an exact modeling of the cancellation mechanism. Due to the complicated rules of the CM, this cannot be analyzed analytically.

3. Conclusion

The revision of the EU ETS, which will come into force completely in 2023, fundamentally changes the system. Due to the introduction of the cancellation mechanism, the aggregate amount of emission permits is endogenous. This implies that national measures and individual efforts that overlap with the EU ETS can have an impact on aggregate total emissions of the associated sectors within the EU. Roughly speaking, the national measure or individual effort leads to a reduction of aggregate emissions if it enhances aggregate holdings of emission permits.

In a purely analytical framework, I derived sufficient conditions so that overlapping national measures have an emission reducing effect. These conditions do hold under the commonly assumed functional form assumptions in the literature that calibrates the future emission path for the EU ETS. Moreover, I investigated the role of low interest rates for the workings of the EU ETS, and, in particular, on the effectiveness of the cancellation mechanism. While a low interest rate increases the aggregate bank and therefore may reduce accumulated emissions, a low interest rate reduces the impact of overlapping national measures.

The applied model is a stylized and extremely simplified description of the EU ETS. In particular, firms in the model operate with a fixed technology. A purpose of the MSR and the CM is to avoid that the carbon price is rather low and the surplus of allowances is high for a prolonged time (European Commission, 2017). If this is the case, the EU ETS does not deliver the necessary investment signal. As this paper shows, the low interest rates of the Euro zone may cause only a slow increase in the carbon price. Therefore, firms have only weak incentives to invest in carbon-free technologies. Moreover, overlapping national measures further reduce the carbon price and thus may also reduce the investment incentives.

Appendix A. Mathematical Appendix

A.1. **Proofs.**

Proof of Proposition 1. Inserting (16) into (17) yields

$$\frac{db(t)}{dM} = \frac{1}{M^2}\left\{\int_0^T S(z)dz \, \frac{\int_0^t h'(w_0 e^{rz})e^{rz}dz}{\int_0^T h'(w_0 e^{rz})e^{rz}dz} - \int_0^t S(z)dz\right\}. \tag{A.1}$$

Note that $db(t=0)/dM = db(t=T)/dM = 0$. Differentiating (17) with respect to t once and twice yields,

$$\dot{\left(\frac{db}{dM}\right)} = -\frac{1}{M^2}S(t) + h'(w_0 e^{rt})e^{rt}\frac{dw_0}{dM}, \text{ and} \tag{A.2}$$

$$\ddot{\left(\frac{db}{dM}\right)} = -\frac{1}{M^2}\dot{S}(t) + \frac{dw_0}{dM} r e^{rt}\Big[h'(w_0 e^{rt}) + h''(w_0 e^{rt})w_0 e^{rt}\Big] > 0, \tag{A.3}$$

respectively. The second derivative with respect to time is positive under the assumptions imposed by Proposition 1. Thus, $db(t)/dM$ is a strictly convex function in t that equals 0 for $t = 0$ and $t = T$. This implies that $db(t)/dM < 0$ for all $t \in (0, T)$. □

Proof of Proposition 2. Differentiation of (20) with respect to M and using (16) leads to

$$\text{(A.4)} \qquad \frac{dB(t)}{dM} = \int_0^t \left\{ -u + h'(w_0 \mathrm{e}^{rz}) + \frac{1}{M} \int_0^T S(\tau) d\tau \, \frac{h'(w_0 \mathrm{e}^{rz}) \mathrm{e}^{rz}}{\int_0^T h'(w_0 \mathrm{e}^{r\tau}) \mathrm{e}^{r\tau} d\tau} \right\} dz.$$

Note that $u - h(w_0 \mathrm{e}^{rt}) = y(t)$. Moreover, $\int_0^T y(t)dt = \int_0^T x(t)dt = \frac{1}{M} \int_0^T S(t)dt$. Hence, $dB(t = 0)/dM = dB(t = T)/dM = 0$. Differentiating (A.4) once and twice with respect to t yields

$$\text{(A.5)} \qquad \frac{d\dot{B}}{dM} = -u + h(w_0 \mathrm{e}^{rt}) + M \frac{dw_0}{dM} h'(w_0 \mathrm{e}^{rt}) \mathrm{e}^{rt} \quad \text{and}$$

$$\text{(A.6)} \qquad \frac{d\ddot{B}}{dM} = r\mathrm{e}^{rt} \left\{ w_0 h'(w_0 \mathrm{e}^{rt}) + M \frac{dw_0}{dM} \left[h'(w_0 \mathrm{e}^{rt}) + h''(w_0 \mathrm{e}^{rt}) w_0 \mathrm{e}^{rt} \right] \right\},$$

respectively. For $h'(\alpha) + \alpha h''(\alpha) \geq 0$ the term in curly brackets is strictly positive. Thus, dB/dM is a strictly convex (concave) function in t for $r > 0$ ($r < 0$). The result now follows from the fact that dB/dM equals 0 at time $t = 0$ and $t = T$. □

Proof of Proposition 3. The signs of the derivatives crucially depend on the sign of the derivative of $(\mathrm{e}^{rt} - 1)/(\mathrm{e}^{rT} - 1)$ with respect to r, which is

$$\text{(A.7)} \qquad \frac{(t - T)\mathrm{e}^{r(t+T)} - t\mathrm{e}^{rt} + T\mathrm{e}^{rT}}{(\mathrm{e}^{rT} - 1)^2}$$

The derivative is negative if the numerator is negative, which is equivalent to

$$\text{(A.8)} \qquad (t - T)\mathrm{e}^{rt}\mathrm{e}^{rT} - t\mathrm{e}^{rt} + T\mathrm{e}^{rt}\mathrm{e}^{r(T-t)} < 0.$$

The above inequality is equivalent to – obtained by rearranging and multiplying both sides with $1/r$:

$$\text{(A.9)} \qquad \frac{\mathrm{e}^{rT} - 1}{rT\mathrm{e}^{rT}} < \frac{\mathrm{e}^{rt} - 1}{rt\mathrm{e}^{rt}}.$$

The above inequality holds if $f(x) := (\mathrm{e}^x - 1)/(x\mathrm{e}^x)$ is a strictly decreasing function in $x > 0$, since $0 < rt < rT$. The derivative of $f(\cdot)$ is

$$\text{(A.10)} \qquad f'(x) = \frac{\mathrm{e}^x(x\mathrm{e}^x) - (\mathrm{e}^x - 1)(\mathrm{e}^x + x\mathrm{e}^x)}{(x\mathrm{e}^x)^2}$$

It holds that $f'(x) < 0$ iff the numerator is negative, i.e., iff

$$\text{(A.11)} \qquad x\mathrm{e}^x - (\mathrm{e}^x - 1)(1 + x) < 0$$

$$\text{(A.12)} \qquad \Longleftrightarrow 1 + x < \mathrm{e}^x.$$

The final inequality holds for $x > 0$ by the properties of the e-function.

From differentiating (25) with respect to r, I obtain that

$$\frac{dB}{dr} = M\frac{(t-T)\mathrm{e}^{r(t+T)} - t\mathrm{e}^{rt} + T\mathrm{e}^{rT}}{(\mathrm{e}^{rT}-1)^2}\left[uT + \frac{aT^2}{2M} - \frac{S_0 T}{M}\right] < 0. \tag{A.13}$$

Taking the derivative of (26) with respect to r yields

$$\frac{d^2B}{drdM} = uT\frac{(t-T)\mathrm{e}^{r(t+T)} - t\mathrm{e}^{rt} + T\mathrm{e}^{rT}}{(\mathrm{e}^{rT}-1)^2} < 0. \tag{A.14}$$

□

A.2. **Sufficient Conditions.** First, we will show that (6)–(9) are not only necessary but also sufficient for optimality. Sufficient conditions for problems of optimal control with pure state constraints are given in Theorem 10.7.1 of Sydsæter, Hammond, Seierstad, and Strøm (2008).

First, $(y^*(t), x^*(t))$ have to maximize $\mathcal{H}(y, x, p(t), t)$. The equations (6) and (7) correspond to the first-order conditions of optimality: $\partial\mathcal{H}/\partial y = 0$ and $\partial\mathcal{H}/\partial x = 0$. Moreover, $\mathcal{H}(y, x, p(t), t)$ is (weakly) concave in (y, x). Next, the theorem requires that $\lambda(t) \geq 0$ and $\lambda(t) = 0$ if $b^*(t) > 0$. This is condition (9). The change in the co-state variable is given by $\dot{p}(t) = -\partial\mathcal{L}^*/\partial b$, which corresponds to (8). At the final point in time T the bank is depleted and thus $b^*(T) = 0$, which implies that there is no condition regarding $p(T)$. Finally, $\mathcal{H}(y^*(t), x^*(t), p(t), t)$ has to be concave in b and the left-hand side of (C) has to be quasi-concave in b. These conditions are obviously satisfied.

Now, I derive sufficient conditions so that $b^*(t) > 0$ for all $t \in (0, T)$. If the constraint (C) is slack, $\lambda(t) = 0$ and the price path evolves according to $\dot{w}(t)/w(t) = r$. If, on the other hand, $b(t) = 0$, then $\dot{p}(t) = -\lambda(t) \leq 0$. From differentiating (7) with respect to t I obtain that

$$\frac{\dot{w}(t)}{w(t)} = r + \frac{\dot{p}(t)}{w(t)} = r - \frac{\lambda(t)}{w(t)} \leq r. \tag{A.15}$$

Thus, a necessary condition for (C) to be binding is that the price increases by less than r. If the bank is depleted, then – in equilibrium – it holds that

$$y(t) = x(t) = \frac{1}{M}S(t). \tag{A.16}$$

By (6) and (7) it has to hold that

$$c'(u - S(t)/M) \equiv w(t). \tag{A.17}$$

Differentiation with respect to t yields

$$-c''(u - S(t)/M)\frac{1}{M}\dot{S}(t) = \dot{w}(t), \tag{A.18}$$

which – using (A.17) – is equivalent to

$$\text{(A.19)} \qquad \frac{c''(u - S(t)/M)}{c'(u - S(t)/M)} \left(\frac{-\dot{S}(t)}{M} \right) = \frac{\dot{w}(t)}{w(t)}.$$

Thus, a sufficient condition for $b(t) > 0$ is that

$$\text{(A.20)} \qquad \frac{c''(u - S(t)/M)}{c'(u - S(t)/M)} \left(\frac{-\dot{S}(t)}{M} \right) \geq r \quad \forall\, t \in (0, T).$$

If $c'''(\cdot) \leq 0$ and $|\dot{S}(T)| \leq |\dot{S}(t)|$ for all t, then (A.20) can be simplified to

$$\text{(A.21)} \qquad \frac{c''(u - S(T)/M)}{c'(u - S(T)/M)} \left(\frac{-\dot{S}(T)}{M} \right) \geq r.$$

For the example discussed in Subsection 2.4, the above condition corresponds to

$$\text{(A.22)} \qquad \frac{1}{Mu - S_0 + aT} \geq \frac{r}{a}.$$

If the interest rate is sufficiently low compared to the (linear) reduction factor, then the bank is always strictly positive (except for $t = 0$ and $t = T$).

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