Do Zombies Rise when Interest Rates Fall? A Relationship Banking Model

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Abstract: A relationship bank or market investors finance an entrepreneur's risky project. Different from investors, the bank can identify and liquidate bad projects at an interim stage. If the entrepreneur can provide only limited capital, the optimal loan contract induces an inefficient continuation decision, i.e., the bank engages in zombie lending. In the short run – for a given contract – the bank's incentive to roll over bad loans enhances if the base interest rate drops. In the long run, however, the bank adjusts the contract to a drop in the interest rate and the effect on zombification is reversed.

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1. Introduction

ZOMBIE FIRMS are the walking dead of an economy: unable to cover their debt obligations with current profits but still staggering on. Banks often keep zombie firms alive by extending or granting loans at favorable terms. The term 'zombie lending' was coined by Caballero *et al.* (2008), who analyzes the so-called lost decade in Japan in the 1990s. Early contributions – but also recent ones – investigating zombie lending point out that weak banks may have incentives to roll over (evergreen) loans to non-viable firms instead of realizing the losses (Peek and Rosengren, 2005; Caballero *et al.*, 2008; Storz *et al.*, 2017; Schivardi *et al.*, 2021).

In the aftermath of the Global Financial Crisis (GFC), zombie lending caught renewed interest, partly due to studies published by researchers of the Organisation for Economic Co-operation and Development (OECD) and the Bank for International Settlements (BIS) (Adalet McGowan *et al.*, 2017; Banerjee and Hofmann, 2018). These studies document a high share of zombie firms in several advanced economies. According to the estimates of Banerjee and Hofmann (2018) for 14 advanced economies, the zombie share increased from 2% in the late 1980s to 12% in 2016. Banerjee and Hofmann (2018) attribute this development to reduced financial pressure rooted in worldwide expansionary monetary policies accompanied by low interest rates. The claim that low interest rates constitute favorable conditions for zombie firms is further supported by empirical studies such as De Martiis and Peter (2021) and Banerjee and Hofmann (2021).

Zombie lending and the channel of low interest rates have also attracted public attention (Banerjee and Hofmann, 2021).[1]

> "Years of ultralow interest rates intended to stimulate the economy after each of three 21st-century recessions created the conditions for zombies to proliferate [...] Weak growth prompts the central bank to cut interest rates, which allows zombies to multiply." — *Washington Post, 2020*[2]

[1] Examples are the following publications: Financial Times on February 5, 2020: "How to avoid a corporate zombie apocalypse" https://www.ft.com/content/1d87c9ec-4762-11ea-aeb3-955839e06441; New York Times on June 15, 2019: "When Dead Companies Don't Die" https://www.nytimes.com/2019/06/15/opinion/sunday/economy-recession.html; The Economist on September 26, 2020: "Why covid-19 will make killing zombie firms off harder" https://www.economist.com/finance-and-economics/2020/09/26/why-covid-19-will-make-killing-zombie-firms-off-harder.

[2] "Here's one more economic problem the government's response to the virus has unleashed: Zombie firms." Washington Post, June 23, 2020, https://www.washingtonpost.com/business/2020/06/

> "As many as one in seven UK firms are potentially "under sustained financial strain" and had been able to "stagger on" partly thanks to low interest rates [...]." — *The Guardian, 2020*[3]

While there is evidence regarding the relation between zombie shares in an economy and the interest rate, the precise mechanism of how low interest rates create a favorable environment for zombie firms is not fully understood. On the contrary, the first effect of a drop in the interest rate should be the reduction of interest expenses, and thus the share of zombie firms (Banerjee and Hofmann, 2018). Therefore, we investigate in a theoretical framework how the interest rate affects banks' incentives to roll over loans to non-viable firms. We model one particular zombification channel that is inspired by the recent theoretical explanation of Hu and Varas (2021). In Hu and Varas (2021), continued bank financing enhances an entrepreneur's reputation and sufficiently reputable entrepreneurs obtain cheap market financing in the future. This creates an incentive for the bank to continue projects that turn out to be of low quality sufficiently late, i.e., to engage in zombie lending.

We build a relationship banking model to address the link between banks' incentives to roll over loans to zombie firms and the base (central bank) interest rate. An entrepreneur can choose between bank or market finance for a risky investment project of an ex ante unknown quality. The bank has higher capital costs but can identify the project's quality earlier than the market – at an interim stage. At this stage, the bank can decide whether to liquidate the project or roll over the loan. Rolling over the loan is a positive signal about the project's quality to market investors who may finance the project at the ex post stage.[4] The loan contract between the relationship bank and the entrepreneur specifies (i) the bank's initial outlay and (ii) the ex post repayment. If the entrepreneur has deep pockets,

23/economy-debt-coronavirus-zombie-firms/.

[3] "Zombie firms' a major drag on UK economy, analysis shows." The Guardian, May 6, 2019, https://www.https://www.theguardian.com/business/2019/may/06/zombie-firms-a-major-drag-on-uk-economy-analysis-shows..

[4] Evidence that a recent bank loan is considered a positive signal by public investors is shown by Ma *et al.* (2019). They document that a borrower who recently obtained a private loan receives more favorable terms for its public bond issuance. Similarly, Bittner *et al.* (2021) find that suppliers (falsely) interpret the bank's roll-over decision as a positive signal about the firm's creditworthiness and are willing to extend trade credits. Already James (1987) points out that bank loans are distinct from other forms of financing. He reports that the announcement of a new bank credit triggered a positive reaction in the borrower's stock prices. A classic theoretical argument based on moral hazard that an early bank loan can enhance borrower reputation, allowing them to switch to direct debt issuance, is provided by Diamond (1991).

the contracted repayment induces efficient continuation, i.e., the contract maximizes the joint surplus of the bank and the entrepreneur. If, however, the entrepreneur is effectively cash constrained ex ante, a second-best loan contract with an inefficiently high repayment is signed. With the repayment being too high, some projects that should be liquidated from a welfare perspective are then continued by the bank at the interim stage: The bank engages in zombie lending.

Our main research question is how a change in the interest rate affects the zombie lending mechanism. Note that a decrease in the interest rate leads to cheaper financing, and hence more project qualities should be continued from a welfare point of view. First, we analyze an unanticipated change in the interest rate, i.e., analyzing the effects of an interest rate drop for a given second-best contract. In this case, the bank has an incentive to roll over even more loans, and the probability of zombie lending increases. The rough intuition is that the bank becomes more patient if the interest rate drops, and thus continuing the project and receiving the inefficiently high ex post repayment becomes more attractive. In the long run, the bank adjusts the offered loan contract to interest rate changes. In this scenario, we can show that the probability of zombie lending decreases with a drop in the interest rate. The reason lies in the market investors' increasing willingness to pay for the risky project ex ante if interest rates decrease. This, in turn, forces the bank to make a more favorable loan contract offer to the entrepreneur. As a result, the adapted loan contract specifies a lower ex post repayment which ultimately reduces the bank's incentive to roll over loans of zombie projects.

To gain a better intuition for our main findings, we extend our baseline model by allowing the three agents – the entrepreneur, the bank, and the market investors – to discount future profits at different rates. The more patient the entrepreneur and the bank are and the less patient the investors are, the more projects are continued at the interim stage. Moreover, to link our results to additional empirical findings, we incorporate the bank's capital structure and overall economic conditions in further extensions. While the relationship bank engages in zombie lending irrespective of its capital structure in our baseline model, we show that banks with lower equity share, and thus higher leverage have higher incentives to roll over loans. In addition, we show that the probability of zombie lending increases in the wake of an economic downturn. These findings are in line with empirical observations, e.g. Giannetti and Simonov (2013) and De Martiis and Peter (2021).

The paper is structured as follows. After discussing the related literature in the following paragraphs, we introduce the model in Section 2. In Subsection 2.2 we derive the first-best outcome and provide a clear definition of zombie lending. Next, we investigate the equilibrium outcome in Section 3, providing conditions for zombie lending to occur in equilibrium. Thereafter, in Section 4, we derive comparative static results concerning changes in the interest rate. In Subsection 4.2 we analyze the effects of an interest rate change on the bank's continuation decision for a given and fixed loan contract. In Subsection 4.3 we take contract adjustments into account. We discuss the extensions and robustness of our model in Sections 5 and 6, respectively. Finally, we conclude in Section 7. All proofs are deferred to Appendix A.

Related Literature

The literature on zombie lending starts with Caballero *et al.* (2008) and Peek and Rosengren (2005), who analyze the impact of the Japanese asset price bubble in the 1990s on the banking industry. They highlight that the housing crisis, combined with the international capitalization requirements (Basel capital standards), pressured banks into not writing off loans. The result of the perverse bank incentives to continue lending relationships with otherwise insolvent firms was a prolonged economic stagnation in Japan, featuring depressed market prices and a general misallocation of resources.[5]

Zombie lending gained renewed attention in the aftermath of the GFC and the European debt crisis. Adalet McGowan *et al.* (2017) and Banerjee and Hofmann (2018) document a high share of zombie firms in various developed economies in recent years. Several articles investigate the role of fiscal stimulus and central bank policies on the prevalence of zombification.[6] For instance, Acharya *et al.* (2021a) find that under-capitalized banks which relied heavier on support from the European Central Bank (ECB) increased their zombie lending. Relatedly, investigating the ECB's Outright Monetary Policy (OMT), Acharya *et al.* (2019) document zombie lending for banks that remained undercapitalized post OMT.[7] Closer related to our paper are the empirical contributions investigating

[5]Related articles that investigate the Japanese banking sector are Hoshi (2000), Giannetti and Simonov (2013) and Kwon *et al.* (2015).

[6]The interaction of regulatory forbearance and zombie lending is investigated by Chari *et al.* (2021). Blattner *et al.* (forthcoming) document that capital requirements affect zombie lending, especially by low-capitalized banks.

[7]Zombie lending in the aftermath of the European debt crisis is also documented by Acharya *et al.* (2020). They report that zombie lending led to excess production capacity, which in turn led to significantly higher pressure on prices, and thus lower inflation. Further empirical studies on zombie lending include Gouveia and Osterhold (2018), Andrews and Petroulakis (2019), and Jordà *et al.*

the connection between the base interest rate and zombie lending (Borio, 2018; Banerjee and Hofmann, 2021; De Martiis and Peter, 2021; Blažková and Chmelíková, 2022). For instance, the estimates by Banerjee and Hofmann (2021, p.32) suggest that "the roughly 10 percentage point decline in nominal interest rates across advanced economies since the mid-1980s can account for around 17 percent of the rise in the zombie share [...]". Similarly, De Martiis and Peter (2021) report evidence, suggesting that low short-term interest rates are favorable for zombie firms.[8]

The theoretical literature on zombie lending can be decomposed into two strands. First, the branch that models weakly capitalized banks with limited liability, which have incentives to 'gamble for resurrection' by keeping their insolvent borrowers alive (Bruche and Llobet, 2014; Acharya *et al.*, 2021c). In Bruche and Llobet (2014), banks privately learn the number of bad loans they possess at an interim stage. At that stage, the return of bad loans is uncertain, and thus banks that possess many bad loans have the incentive to hide losses and gamble for resurrection.[9] Bruche and Llobet (2014) propose a regulatory regime that induces banks to disclose their bad loans. Relying on a related explanation for zombie lending, Acharya *et al.* (2021c) build a model with heterogeneous firms and heterogeneous banks. Firms differ in their productivity and risk, whereas banks differ in their equity share. The model gives rise to 'diabolic sorting': poorly capitalized banks lend to firms with low productivity.[10] Acharya *et al.* (2021c) also analyze the impact of conventional (interest rate) and unconventional (forbearance) monetary policy on zombification. They point out that, in a dynamic setting, myopic policies result in low interest rates and high forbearance that keep zombies alive and productivity low. In contrast to our findings, low interest rates alone without forbearance do not promote zombie lending.

Second, and more closely related to our study, is the extant literature that relies on models of relationship banking to explain zombie lending (Faria-e-Castro *et al.*, 2021; Hu and Varas, 2021; Aragon, 2022).[11] Faria-e-Castro *et al.* (2021) develop a model in which relationship banks evergreen loans by offering better credit terms to less productive and

(2022).

[8]In a VOXeu column, Laeven *et al.* (2000) question whether there is a clear link between low interest rates and zombification.

[9]A related model where banks have the incentive to roll over loans to hide the loan quality from the market is analyzed by Rajan (1994).

[10]Tracey (2021) proposes a further model where zombie lending helps low productivity firms to survive.

[11]According to most models, zombie lending has negative implications for the economy. An exception is Jaskowski (2015) who builds a model where zombie lending improves ex ante lending and can prevent ex post fire sales, thereby improving overall efficiency.

more indebted firms. Different from market investors, the relationship bank owns a firm's legacy debt, and thus has the incentive to increase the continuation value of its firm. As a result, financially distressed firms receive 'discounted' credit terms from relationship banks to reduce their probability of default. It follows that relationship banking leads to dispersion in firms' marginal product of capital, and thus an inefficient capital allocation. Moreover, banks' evergreening of loans leads to higher levels of debt and lower aggregate productivity. Aragon (2022) models competition for firm financing between an incumbent bank owning the firm's legacy debt and a competitor. He shows that the firm's debt overhang creates monopoly power for the incumbent bank which may be used to extract larger rents. This rent extraction incentive by the bank may prevent the borrowing firm from investing in a new and on average profitable technology. According to Aragon (2022), zombification describes the situation where a firm that cannot fully repay its outstanding debt is kept alive but is unable to invest, and thus unable to improve its productivity.

Regarding the modeled zombification mechanism, the article closest related to our study is by Hu and Varas (2021). They consider a dynamic continuous time model where an entrepreneur initially chooses between bank finance and market finance. The bank has higher costs of capital but receives private information regarding the quality of the entrepreneur's project over time. The quality of the project is either good or bad. Once the bank (and the entrepreneur) learns that the project is bad, continued financing is costly. However, if the project is financed for sufficiently many periods by the bank, market investors believe that its quality is high, and are thus willing to pay a high price for it.[12] This creates an incentive for the bank to continue projects that turn out to be of bad quality at interim points in time. These projects are sold later to market investors which are 'deceived' by the roll-over decision. While in Hu and Varas (2021) good projects should always obtain financing and bad ones should always be liquidated, the welfare optimal quality threshold is endogenous in our model. In other words, it is optimal to liquidate fewer projects if interest rates are low. Moreover, the implications of interest rate changes on a bank's incentive to engage in zombie lending are not at the heart of Hu and Varas (2021).

[12]Somewhat related, Puri (1999) builds a model where the bank's decision at an intermediate stage affects investor evaluations of securities the bank underwrites. In her model, investors may effectively repay the firm's bank loan.

2. The Model

2.1. Players & Timing

We consider a model with three dates $t = 0, 1, 2$. There are three types of risk-neutral agents: an entrepreneur (she), a relationship bank, and investors. We denote all variables in terms of their respective date $t = 2$ future values.[13]

At $t = 0$, the entrepreneur owns a risky business project of ex ante unknown quality θ. The project requires an initial investment of $I > 0$ at $t = 0$. If the project is initiated at $t = 0$, then it generates a payoff of $\gamma\theta$, with $\gamma > 0$, at the end of date $t = 1$, and a payoff of θ at date $t = 2$. The project quality is distributed according to c.d.f. $F(\theta)$ and density $f(\theta) > 0$ on $[\underline{\theta}, \bar{\theta}]$. The expected quality

$$\mu := \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta > 0 \tag{1}$$

is assumed to be strictly positive. The entrepreneur's initial wealth is $w \geq 0$. We assume that $w < I$ so that the entrepreneur requires external finance to implement her business project. The entrepreneur can sign a loan contract with the bank or lend money from (sell the project to) investors. She can also decide not to implement the business project.

At $t = 0$ the bank can make a take-it-or-leave-it loan contract (d, R) offer to the entrepreneur. The bank finances $I - d$ of the project, and the entrepreneur invests equity capital d. The contract also specifies the gross repayment R from the entrepreneur to the bank at $t = 2$. For ease of exposition, we assume that the contract transfers the date $t = 1$ cash flow and control rights to the bank (instead of specifying next to the final repayment also an interim repayment).[14] At $t = 1$, the bank has the cost of $c > 0$ for engaging in this relationship lending, which can be interpreted as monitoring costs. Due to this monitoring, the bank learns the quality of the project θ at the beginning of date $t = 1$. The bank then decides whether to continue the project or liquidate it. In

[13]From Section 4 onward, we explicitly express the variables' dependence on the interest rate. As an example, suppose the interest rate is $r \geq 0$ and the project requires at $t = 0$ an initial investment of $\tilde{I}$. The date $t = 2$ future value of this investment is $I = (1 + r)^2 \tilde{I}$.

[14]The assumption that at $t = 1$, the bank obtains the project's full return and control rights seems extreme at first glance. An alternative interpretation is that the specified repayments in $t = 1$ exceed the return at that date (at least for the marginal project quality). In this case, the bank can decide whether to extend the loan or not. If the loan is not extended, the project is bankrupt and the bank obtains the liquidation value. In Section 6.1 we consider the case where the loan contract specifies repayments in $t = 1$ and $t = 2$, and the entrepreneur keeps the cash-flow and control rights (as long as she can make the repayment). The results are qualitatively identical.

case of liquidation, the project pays a liquidation value $L > 0$ at the end of date $t = 1$. This liquidation value L is independent of the project's quality θ. A continued project generates a return of $\gamma\theta$ at the end of date $t = 1$ and of θ at date $t = 2$. Finally, the parties commit at $t = 0$ to terminate the relationship at the beginning of $t = 2$ and to sell the project to investors. In other words, the project sell-off to the investors, and thus R, is made before the return θ realizes. Let $L < (1 + \gamma)\mu$.

There is a large group of investors that act in a perfectly competitive financial market. Investors can either purchase (finance) the project at a price P_0 at date $t = 0$ or at a price P_2 at the beginning of date $t = 2$.[15] If investors purchase the project at date $t = 0$, they learn the project's quality only indirectly at the end of date $t = 1$ where it pays out $\gamma\theta$. At this point, it is no longer possible to liquidate the project in $t = 1$ (and there is no liquidation opportunity in $t = 2$). Thus, the disadvantage of market finance compared to bank finance is that projects with low returns can not be terminated at the intermediate date $t = 1$. The advantage of market finance is that the market does not have any costs. If investors purchase the project at the beginning of date $t = 2$, they pay a price P_2 to the entrepreneur and receive the return θ at the end of date $t = 2$. Importantly, if the project is initially financed via the bank, there is asymmetric information at date $t = 2$ between the bank/entrepreneur and investors. The investors do not know the quality of the project. However, they do observe the signed loan contract and correctly understand the bank's incentives to continue projects at date $t = 1$, and thus update their belief regarding the offered project's quality accordingly.

The timeline of our model, in particular the project's investment and returns at the three dates, are depicted in Figure 1.

A few remarks regarding the optimality of the allowed contracts are in order. The loan contracts we are analyzing are optimal under two conditions. First, the parties – bank and entrepreneur – can commit at $t = 0$ to sell the project to investors at the beginning of $t = 2$. This is efficient as the bank has higher operating costs ($c > 0$) than the market and, once θ is learned, there is no benefit from bank monitoring. Ex post, at $t = 2$, the entrepreneur prefers to sell only 'bad' projects and to keep 'good' ones since the return

[15] With all parties being risk-neutral, the assumption that investors purchase the whole project at $t = 0$ is without loss in generality. To see this, suppose the entrepreneur sells shares α of her project to investors to finance $I - w$. The lowest share that investors are willing to accept is $\hat{\alpha} = (I - w)/[(1 + \gamma)\mu]$. The expected profit of the entrepreneur from selling share $\hat{\alpha}$ of the project is $\mathbb{E}[-w + (1 - \hat{\alpha})\gamma\theta + (1 - \hat{\alpha})\theta] = (1 + \gamma)\mu - I$. Moreover, note that risk-neutral investors could also finance the project at the beginning of date $t = 1$. This, however, will never happen in equilibrium because the monitoring cost is sunk at the beginning of $t = 1$ but the liquidation decision (usage of the information) is not yet made.

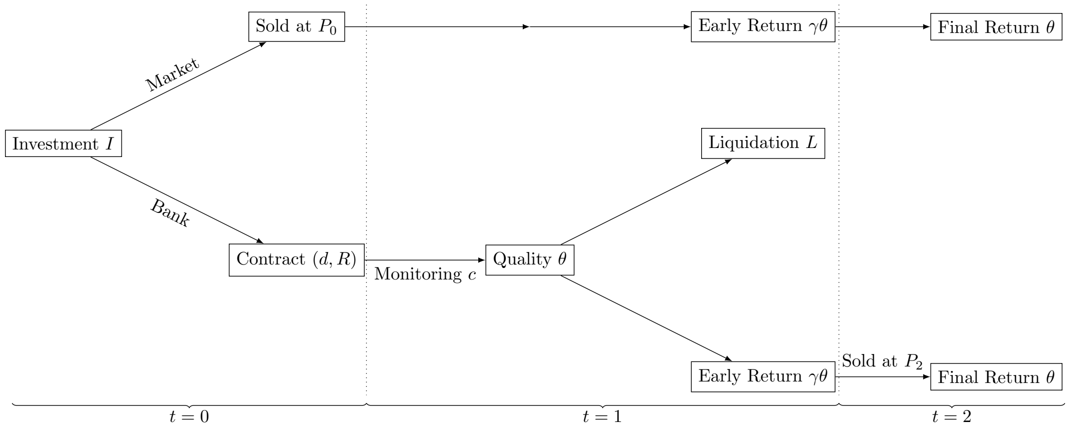

Figure 1: Timeline of the project's investment, liquidation, and returns.

$\theta - R$ accrues to the entrepreneur. The bank, however, does not benefit from not selling 'good' projects to the market. It obtains at most the repayment R. The bank suffers if only bad projects are sold to market investors who anticipate this adverse selection, and thus $R > P_2$. Hence, it is in the bank's interest to have a reputation for sticking to the original contract – selling continued projects to the market at $t = 2$ – instead of engaging in renegotiation with the entrepreneur.

Second, the parties cannot commit to a certain roll-over decision, i.e., it is not feasible to specify a critical quality threshold $\hat{\theta}$ directly in the contract. The idea is that at the beginning of $t = 1$, the quality is privately observed by the bank (and the entrepreneur). An outsider, say a court, cannot observe and verify θ. Thus, the roll-over decision can only be incentivized indirectly via payments to the bank at dates $t = 1$ and $t = 2$.

Finally, note that there is no scope for signaling the project quality by financing parts with own funds in the spirit of Leland and Pyle (1977). Ex ante, at $t = 0$, there is symmetric information between all three types of agents. Ex post, at $t = 2$, the project is always sold to the market – by the assumption that the bank can credibly commit not to renegotiate the contract – and thus there is no inefficiency arising from adverse selection that can be mitigated by signaling (via selling only shares of the project to the market).

2.2. First-best Benchmark and Definition of Zombie Lending

In case of market finance via investors, information is only revealed at the end of date $t = 1$, implying that early liquidation is not optimal. Thus, the expected surplus generated by market finance at $t = 0$ is

$$(1 + \gamma)\mu - I. \tag{2}$$

In case of bank finance, the project's quality is observed at the beginning of date $t = 1$. This allows liquidating low quality projects at date $t = 1$. The continuation of a project is efficient at $t = 1$ if the project's total return is higher than the liquidation value, i.e., if $\gamma\theta + \theta \geq L$. This inequality is equivalent to

$$\theta \geq \frac{L}{1+\gamma} =: \theta^*. \tag{3}$$

We call θ^* the efficient quality threshold. The efficient quality threshold θ^* is increasing in the liquidation value L and decreasing in the $t = 1$ share of the project's return γ. The expected surplus generated by efficient bank financing is

$$\int_{\underline{\theta}}^{\bar{\theta}} \max\{(1+\gamma)\theta, L\} f(\theta)\, \mathrm{d}\theta - c - I. \tag{4}$$

The following result summarizes the first-best outcome.

Observation 1 (First-best Finance)**.** *In the first-best situation, the project is*

(i) financed by the bank if $c \leq \bar{c}^{FB}$*;*

(ii) financed by investors (financial market) if $c > \bar{c}^{FB}$ *and* $I \leq (1+\gamma)\mu$*;*

(iii) not financed in all remaining cases.

The threshold value for the monitoring cost is

$$\bar{c}^{FB} := \begin{cases} \int_{\underline{\theta}}^{\bar{\theta}} \max\{(1+\gamma)\theta, L\} f(\theta)\, \mathrm{d}\theta - (1+\gamma)\mu & \text{for } I \leq (1+\gamma)\mu, \\ \int_{\underline{\theta}}^{\bar{\theta}} \max\{(1+\gamma)\theta, L\} f(\theta)\, \mathrm{d}\theta - I & \text{for } I > (1+\gamma)\mu. \end{cases} \tag{5}$$

Note that $\bar{c}^{FB} > 0$ for $I < (1+\gamma)\mu$.

Having characterized the first-best outcome and in particular the first-best continuation decision of the bank, we are now in the position to define zombie lending.

Definition 1 (Zombie Lending)**.** *If at date $t = 1$ the bank continues a project (rolls over the credit) of quality less than the efficient threshold, $\theta < \theta^*$, we define this as zombie lending.*

According to our definition, zombie lending occurs if a project is not liquidated even though liquidation maximizes the generated surplus.

3. Financing Analysis

3.1. Bank's Optimization Problem

Suppose the bank and the entrepreneur can sign a contract that is profitable for both parties. The loan contract (d, R) offered by the bank maximizes its expected profit

$$\pi_B(d, R) = F(\hat{\theta}(R))L + \gamma \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta + [1 - F(\hat{\theta}(R))]R - c - I + d \tag{6}$$

subject to

$$\pi_E(d, R) \geq \max\{P_0, 0\}, \tag{PC}$$

$$\hat{\theta}(R) = \frac{1}{\gamma}(L - R), \tag{RD}$$

$$d \leq w \tag{LL}$$

where

$$\pi_E(d, R) = -d + [1 - F(\hat{\theta}(R))][P_2(\hat{\theta}(R)) - R]. \tag{7}$$

denotes the entrepreneur's net expected profit.

The offer made by the bank is accepted by the entrepreneur only if the participation constraint (PC) is satisfied. Recall that there is a large number of risk-neutral investors. At date $t = 0$, these investors are willing to pay

$$P_0 := (1 + \gamma)\mu - I \tag{8}$$

for a project of unknown quality. At $t = 2$ investors update their quality expectation taking the bank's roll-over decision into account, and are thus willing to pay

$$P_2(\hat{\theta}(R)) = \mathbb{E}[\theta \mid \theta \geq \hat{\theta}(R)] = \frac{1}{1 - F(\hat{\theta}(R))} \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta. \tag{9}$$

Moreover, the bank considers how the signed loan contract (d, R) affects its own roll-over decision, constraint (RD). The bank rolls over the loan at $t = 1$ if and only if

$$\gamma\theta + \min\{R, P_2 + w - d\} \geq L. \tag{10}$$

In case of roll-over, the bank obtains $\gamma\theta$ at the end of date $t = 1$ and the repayment R at date $t = 2$. If, however, the entrepreneur cannot repay R, then the entrepreneur is bankrupt and the bank obtains her remaining capital, $P_2 + w - d$. Note that it does not make sense to specify a repayment that can never be made by the entrepreneur. Thus, we can focus on $\min\{R, P_2 + w - d\} = R$, and the bank continues all projects with qualities $\theta \geq \hat{\theta}(R)$.

Finally, the entrepreneur's initial outlay cannot exceed her wealth, i.e., the limited liability constraint (LL) must hold.

3.2. Optimal Loan Contract

Note that the amount initially invested by the entrepreneur herself, d, is an ex ante one-to-one transfer between the bank and the entrepreneur. A higher d increases the bank's expected profit, does not affect the bank's roll-over decision, and the joint surplus of the bank and entrepreneur is independent of d. Thus, if constraint (LL) is slack, the bank has the incentive to offer a loan contract that maximizes the rents generated by bank finance. These rents are maximized if and only if the roll-over decision is efficient. The bank makes an efficient roll-over decision if and only if $\hat{\theta}(R) = \theta^*$. This is achieved for the repayment

$$R^* = \frac{L}{1+\gamma} = \theta^*. \tag{11}$$

Let d^* be the entrepreneur's initial outlay that satisfies the participation constraint with equality for $R = R^*$, implicitly given by $\pi_E(d^*, R^*) = \max\{(1+\gamma)\mu - I, 0\}$. Given that the entrepreneur's initial outlay can not exceed her wealth, $d \leq w$, the first-best loan contract (d^*, R^*) is feasible, and thus offered if $d^* \leq w$.

Proposition 1 (First-best Contract)**.** *Suppose bank lending is efficient. Then, the loan contract (d, R) offered by the bank induces the efficient roll-over decision at $t = 1$ if*

$$w \geq \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\, \mathrm{d}\theta - \max\{P_0, 0\} =: d^*. \tag{12}$$

The loan contract specifies

$$d = d^* \text{ and } R = R^* = \theta^*. \tag{13}$$

If the entrepreneur does not have sufficiently deep pockets, $w < d^*$, the bank cannot extract the full additional surplus that is generated by efficient bank lending. The bank will specify the highest feasible initial outlay by the entrepreneur, i.e., $d = w$. In this case, the bank faces a tradeoff between rent extraction and efficiency. The bank can increase its expected profit by increasing the repayment R above the efficient level $R^* = \theta^*$. This, however, distorts the continuation decision at date $t = 1$. The bank continues a project if the quality θ is above $\hat{\theta}(R) = \gamma^{-1}(L - R)$, with $d\hat{\theta}/dR = -\gamma^{-1} < 0$. Note that for R^* it holds that $\hat{\theta}(R^*) = \theta^*$. Thus, for $R > R^*$ it holds that $\hat{\theta} < \theta^*$. The financial market anticipates the bank's lenient roll-over decision, and thus reduces its willingness to pay for the project at date $t = 2$. There is a maximum feasible repayment $\bar{R}$, implicitly defined by

$$\mathbb{E}[\theta \mid \theta \geq \hat{\theta}(\bar{R})] = \bar{R}. \tag{14}$$

Note that $\bar{R} > R^*$. Inserting $d = w$ and $R = \bar{R}$ into the entrepreneur's expected profit (7) yields $\pi_E = -w$. All repayments $R > \bar{R}$ violate the participation constraint (PC). The expected profit of the bank $\pi_B(d = w, R)$ is strictly increasing in the repayment $R \leq \bar{R}$ with

$$\frac{\partial \pi_B}{\partial R} = 1 - F(\hat{\theta}) > 0. \tag{15}$$

This implies that the bank specifies the highest repayment that the entrepreneur is just willing to accept, i.e., the repayment that makes the entrepreneur indifferent between the offered bank loan and her best alternative option.

Proposition 2 (Second-best Contract)**.** *Suppose $w < d^*$ and that the bank can make a profitable offer that is accepted by the entrepreneur. Then, the bank offers the second-best optimal loan contract (d^{SB}, R^{SB}), with $d^{SB} = w$ and $R^{SB} \in (\theta^*, \bar{R}]$ implicitly defined by $\pi_E(d^{SB}, R^{SB}) = \max\{P_0, 0\}$.*

If the entrepreneur is effectively cash constrained but bank finance nevertheless occurs in equilibrium, then a loan contract is signed with a too high repayment $R^{SB} > R^*$ from an efficiency point of view. Thus, the bank rolls over projects with quality below the efficient quality threshold θ^*. In other words, the bank engages in zombie lending, depicted in Figure 2.

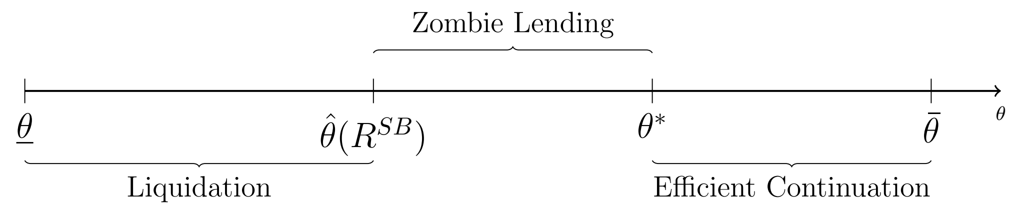

Figure 2: The bank's decision at date $t = 1$ under a second-best contract.

Corollary 1. *Under the second-best loan contract (d^{SB}, R^{SB}) zombie lending takes place for projects of quality $\theta \in [\hat{\theta}(R^{SB}), \theta^*)$.*

This is a very important observation: In case the entrepreneur is effectively cash-constrained, $w < d^*$, there is scope for (inefficient) zombie lending. The parameters for which zombification occurs in equilibrium are analyzed in the next section.

3.3. Equilibrium Finance

Now, we analyze which form of financing occurs in equilibrium. In particular, we investigate the conditions in which the entrepreneur and the bank sign the second-best loan contract in equilibrium. We depict the findings in Figure 3: the horizontal axis scales the investment I and the vertical axis the monitoring cost c.

On the one hand, market finance is only feasible if the initial investment is not too high,

$$I \leq (1+\gamma)\mu. \tag{16}$$

On the other hand, first-best bank financing leads to a higher expected surplus than market financing if the monitoring cost is rather low, $c \leq \bar{c}^{FB}$ (see Observation 1). The bank offers the first-best contract (d^*, R^*) only if the entrepreneur possesses sufficient initial wealth, i.e., if $w \geq d^*$. For $I \leq (1+\gamma)\mu$, and thus $P_0 \geq 0$, the condition $w \geq d^*$ is equivalent to

$$I \leq (1+\gamma)\mu + w - \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\, \mathrm{d}\theta =: \bar{I}^{FB}. \tag{17}$$

For projects with low initial financing volume, $I \leq \bar{I}^{FB}$ (and $c < \bar{c}^{FB}$), the bank offers the first-best contract. In case of higher required initial investments, the bank either offers the second-best contract or no contract.

A priori, it is not clear whether the critical threshold $\bar{I}^{FB}$ is smaller or larger than $(1+\gamma)\mu$. In the following, we focus on the former case, which applies if the entrepreneur's initial wealth is not too large. In this regard, we impose

Assumption 1. *The entrepreneur's initial wealth is lower than the expected surplus generated by efficient continuation:*

$$w < \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\, \mathrm{d}\theta. \tag{18}$$

The bank offers the second-best contract, where $d^{SB} = w$ and R^{SB} is determined by the participation constraint, only if its profit $\pi_B(w, R^{SB})$ from the contract is non-negative. The second-best repayment is determined by $\pi_E(w, R^{SB}) = \max\{(1+\gamma)\mu - I, 0\}$, and thus is a function of the initial investment I but is independent of the monitoring cost c. Formally, $R^{SB} = R^{SB}(I)$. The expected profit of the bank from offering contract $(d^{SB} = w, R^{SB}(I))$ is non-negative if and only if $c \leq \bar{c}^{SB}(I)$, where

$$\begin{aligned} \bar{c}^{SB}(I) \equiv F(\hat{\theta}(R^{SB}(I)))L + \gamma \int_{\hat{\theta}(R^{SB}(I))}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta \\ + [1 - F(\hat{\theta}(R^{SB}(I)))]R^{SB}(I) - I + w. \end{aligned} \tag{19}$$

Equation (19) defines the cost threshold as a function of the initial investment I. Importantly, for $I \searrow \bar{I}^{FB}$ it holds that $\bar{c}^{SB}(I) \to \bar{c}^{FB}$.[16] The critical threshold of the monitoring cost $\bar{c}^{SB}(I)$ is a strictly decreasing function in I. For large initial investments I, the threshold $\bar{c}^{SB}$ is negative which implies that second-best bank finance is not profitable.

The equilibrium contracts are summarized in the following proposition.

Proposition 3 (Equilibrium Finance)**.** *Suppose that Assumption 1 holds. Then, the date $t = 0$ equilibrium decision of the entrepreneur is*

(i) market finance if and only if

$$c \geq \begin{cases} \bar{c}^{FB} & \text{for } I \leq \bar{I}^{FB}, \\ \bar{c}^{SB}(I) & \text{for } I \in (I^{FB}, (1+\gamma)\mu]; \end{cases} \tag{20}$$

(ii) bank finance if and only if

$$c < \begin{cases} \bar{c}^{FB} & \text{for } I \leq \bar{I}^{FB}, \\ \bar{c}^{SB}(I) & \text{for } I > I^{FB}; \end{cases} \tag{21}$$

(iii) no finance in all other cases.

As shown in Figure 3, the project is not financed at all if the initial investment is too large. A project with a low or moderately high initial investment is financed in equilibrium. Such a project is financed by the financial market (sold to investors at date $t = 0$) if the bank's monitoring cost is high, otherwise, it is initially financed with a bank loan. The bank offers the first-best contract if the initial investment is low, $I \leq \bar{I}^{FB}$. In this case, bank finance is efficient. For moderately high initial investments, $I \in (\bar{I}^{FB}, (1+\gamma)\mu]$, and low monitoring cost, $c \leq \bar{c}^{SB}$, the bank offers the second-best contract. In this case, first-best bank lending is efficient but the equilibrium outcome is second-best bank lending with a distorted continuation decision. Moreover, for $I \in (\bar{I}^{FB}, (1+\gamma)\mu]$ and $c \in (\bar{c}^{SB}, \bar{c}^{FB})$ first-best bank lending is efficient but in equilibrium, the project is financed by the financial market. Finally, for some projects with $I > (1+\gamma)\mu$ the efficient outcome is bank finance. In equilibrium, however, these projects are either not financed at all or with a second-best loan contract offered by the bank.

In summary, three distortions may arise in equilibrium: First, a project with a strictly positive expected net return from efficient bank lending is not financed in equilibrium

[16]To see this formally, note that for $I = \bar{I}^{FB}$ we have $R^{SB} = R^* = \theta^*$ and $\hat{\theta} = \theta^*$. Solving $\pi_E(d = w, R^{SB}) = \max\{P_0, 0\}$ for w and inserting this into (19) yields the result.

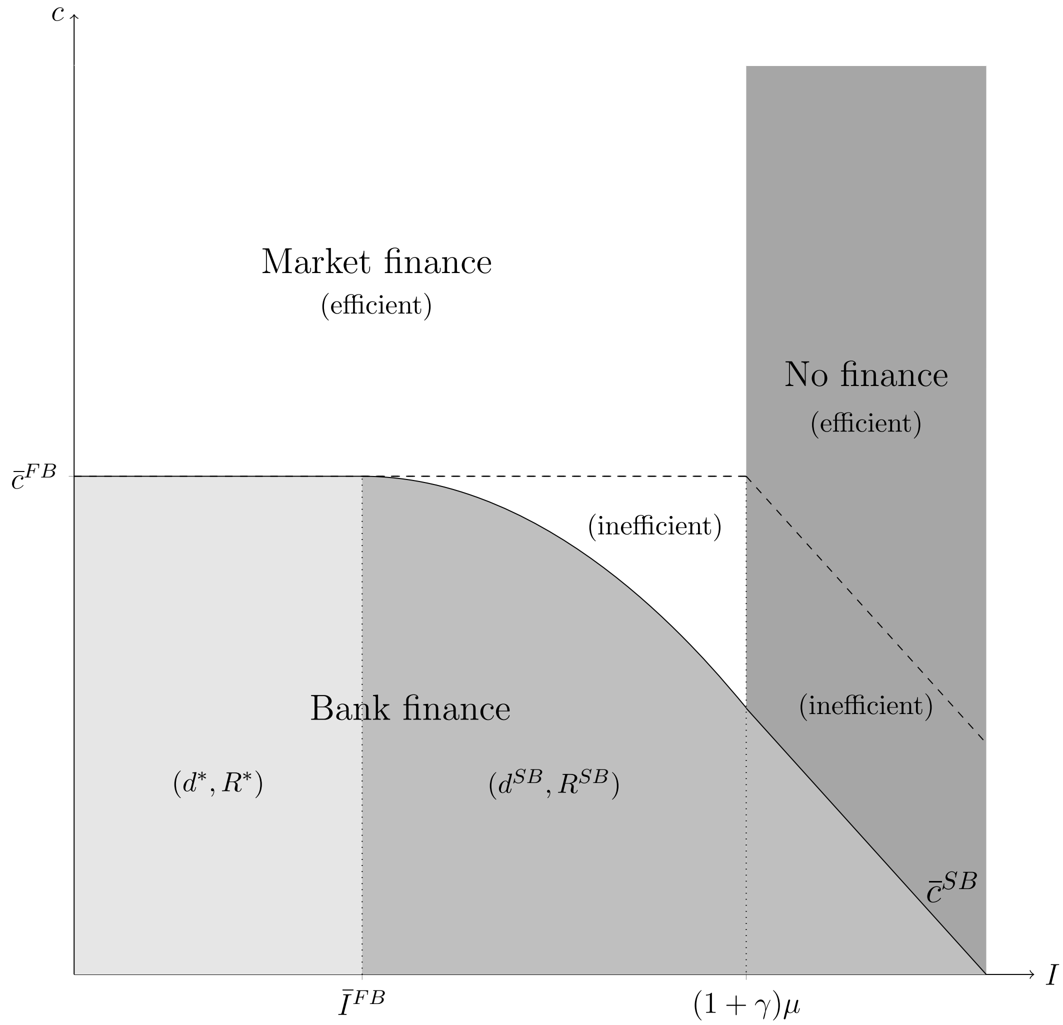

Figure 3: Equilibrium finance and efficiency

(credit crunch). Second, a project that – from a welfare perspective – should be financed by the bank, is financed by investors in equilibrium (inefficient financing form). Third, a project is financed via the second-best loan contract rather than efficient bank lending, which creates incentives for zombie lending.

The focus of our paper is on the third inefficiency, zombie lending. The following result summarizes the conditions so that zombie lending – inefficient roll-over decisions – occur in equilibrium.

Corollary 2 (Zombie Lending)**.** *Suppose that Assumption 1 holds. In equilibrium, the bank and the entrepreneur sign a second-best loan contract* (d^{SB}, R^{SB}) *if and only if* $I > \bar{I}^{FB}$ *and* $c < \bar{c}^{SB}(I)$*. In this case, the bank engages in the zombification of projects of quality* $\theta \in [\hat{\theta}(R^{SB}), \theta^*)$*.*

4. Interest Rates and Zombification

4.1. Research Question and Notation

Besides explaining the occurrence of zombie lending, we are particularly interested in how a change in the interest rate affects zombie lending. We assume that all agents – the entrepreneur, the bank, and the investors – discount future payments based on an identical interest rate $r \geq 0$. This interest rate can be interpreted as being determined, albeit only indirectly, by the policy of a central bank.[17]

As explained in Section 2, all variables can be interpreted as the date $t = 2$ future value of the respective variable. We denote the actual numerical value of each variable with a tilde. Thus, we can introduce the following variable transformation:

$$\begin{aligned} \gamma &= (1+r)\tilde{\gamma}, \quad & c &= (1+r)\tilde{c}, \\ L &= (1+r)\tilde{L}, \quad & I &= (1+r)^2\tilde{I}, \\ w &= (1+r)^2\tilde{w}, \quad & d &= (1+r)^2\tilde{d}. \end{aligned}$$

Note that variables occurring at date $t = 2$ need no transformation, e.g. the repayment still denotes R.

We are interested in how a change in the interest rate affects a bank's decision to roll over credit. Therefore, we focus on the financing scenario where the entrepreneur and the bank sign a second-best loan contract (d^{SB}, R^{SB}).

The efficient roll-over quality threshold is

$$\theta^*(r) = \frac{(1+r)\tilde{L}}{1+(1+r)\tilde{\gamma}}. \tag{22}$$

A change in the interest rate affects the efficient quality threshold as follows:

$$\frac{d\theta^*}{dr} = \frac{\tilde{L}}{[1+(1+r)\tilde{\gamma}]^2} > 0. \tag{23}$$

Thus, if the interest rate decreases, it is welfare optimal to roll over more loans. This is intuitive because a lower interest rate makes the date $t = 2$ project return θ relatively more important than the date $t = 1$ project liquidation value $\tilde{L}$. In other words, the continuation decision is cheaper if the interest rate decreases.

[17] Investigating the optimal central bank policy is outside the scope of this paper. The central bank may induce an interest rate that seems inefficient from our model's point of view because it takes other reasons that are not modeled here into account.

Under a second-best loan contract, the bank rolls over all loans of quality θ weakly larger than

$$\hat{\theta}(r, R^{SB}) = \frac{(1+r)\tilde{L} - R^{SB}}{(1+r)\tilde{\gamma}}. \tag{24}$$

In the following, we consider two scenarios. First, we investigate the effects of changes in the interest rate for a given loan contract (short-run analysis). Thereafter, we take the impact of a change in the interest rate on the offered contract into account.

4.2. Short-run Effects of Interest Rate Changes

As a first step, we investigate the effect of an adjustment in the interest rate r on the probability of zombie lending

$$Z(r) = Prob(\theta \in [\hat{\theta}, \theta^*)), \tag{25}$$

for a given second-best loan contract (d^{SB}, R^{SB}). This effect can be interpreted as the effect of an unanticipated change in the interest rate. Namely, the entrepreneur and the bank signed a second-best loan contract at date $t = 0$. At the beginning of date $t = 1$, the interest rate changes and this change was not expected by the bank or the entrepreneur. Thus, at date $t = 1$ the contract is given but the bank can adjust its roll-over decision. If the interest rate increases, the bank applies a stricter roll-over rule, i.e.,

$$\frac{\partial \hat{\theta}}{\partial r} = \frac{R^{SB}}{\tilde{\gamma}(1+r)^2} > 0. \tag{26}$$

The intuition is analog to the efficient threshold argument. To obtain a clear-cut finding in this section, we assume the following:

Assumption 2. *For all $\theta \in [\underline{\theta}, \bar{\theta}]$ it holds that $f'(\theta) \leq 0$.*

According to Assumption 2, projects of higher quality are less likely, i.e., 'unicorns' are rare. We are then able to make the following proposition.

Proposition 4. *Suppose that Assumption 2 holds and that the entrepreneur and the bank signed a second-best loan contract. Then, an unanticipated reduction in the interest rate increases the probability of zombie lending, i.e.,*

$$Z(r) = \int_{\hat{\theta}(r, R^{SB})}^{\theta^*(r)} f(\theta)\, \mathrm{d}\theta \tag{27}$$

is strictly decreasing in r.

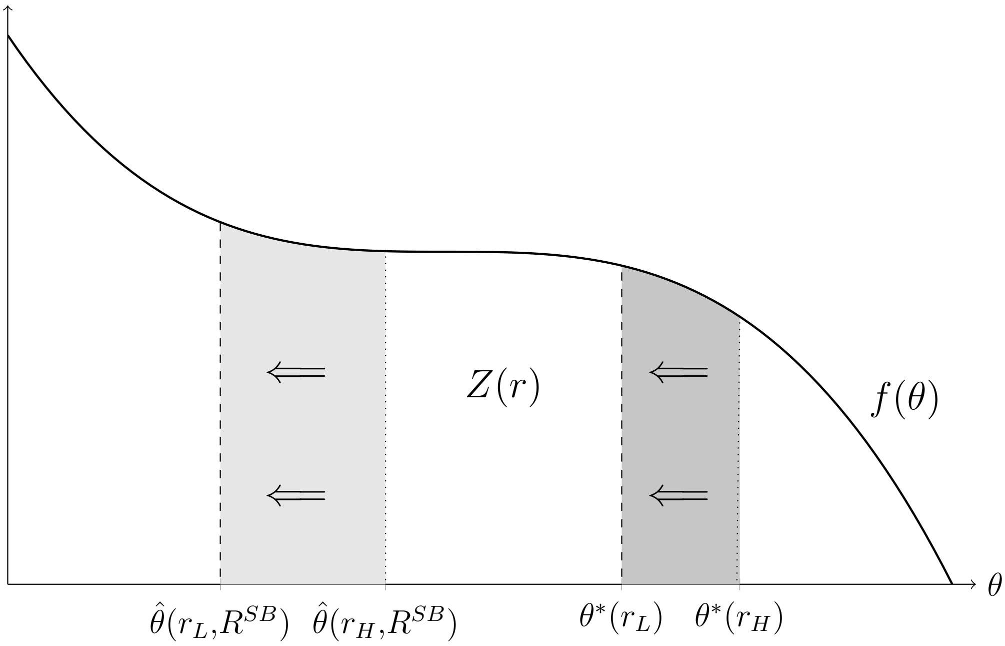

Figure 4: The bank's adjusted roll-over decision for an unexpected drop in interest rates from r_H to $r_L < r_H$.

Proposition 4 states that if the entrepreneur and the bank engage in a long-term lending relationship and during this relationship the interest rate drops unexpectedly, then the bank rolls over even more loans compared to the efficient continuation decision.

As highlighted in Figure 4, the probability of zombie lending $Z(r)$ increases with decreasing interest rates r for any density function $f(\theta)$, with $f'(\theta) \leq 0$. Note that any drop (rise) in the interest rate r increases (decreases) the zombie lending interval, $\theta \in [\hat{\theta}(r), \theta^*(r))$. Specifically, the mass of qualities θ in the interval of $\hat{\theta}(r_L, R^{SB})$ and $\hat{\theta}(r_H, R^{SB})$ is strictly larger than the corresponding mass in the interval of $\theta^*(r_L)$ and $\theta^*(r_H)$, for $r_H > r_L$. Conveying the result to the real world, this scenario may very well resemble many lending relationships between commercial banks and companies following the financial crises in the EU, i.e., in the early 2010s. Thus, according to our theory, the – to some degree – unexpected continued loose monetary policy of the ECB after the financial crises may have augmented the problem of zombie lending in the euro area.

Proposition 4 also has implications regarding the probability of zombie lending under a formerly first-best contract. Under the first-best contract, the repayment is $R^*(r) = \theta^*(r)$ so that the bank applies the efficient quality threshold $\hat{\theta}(r, R^*(r)) = \theta^*(r)$. Now, suppose the interest rate drops from r_H to $r_L < r_H$. This decreases the first-best threshold from $\theta^*(r_H)$ to $\theta^*(r_L)$. Given that the interest rate drop was unexpected, the repayment stays at

$R^*(r_H)$ while the bank applies the quality threshold $\hat{\theta}(r_L, R^*(r_H))$. It can readily be shown that $\hat{\theta}(r_L, R^*(r_H)) < \theta^*(r_L)$, and thus zombie lending occurs for qualities $\theta \in [\hat{\theta}, \theta^*)$. In other words, an unanticipated drop in the interest rate also increases the scope for zombie lending under the formerly first-best loan contract (d^*, R^*).[18]

4.3. Long-run Effects of Interest Rate Changes

In this section, we assume that the interest rate changes before the parties sign a loan contract. We remain in the scenario where the entrepreneur and the bank sign a second-best loan contract. We investigate how this loan contract adapts to a change in the interest rate. In particular, we are interested in how the repayment $R^{SB} = R^{SB}(r)$ adjusts and how this affects the bank's roll-over decision at $t = 1$. Under the second-best contract, the amount financed by the entrepreneur $\tilde{d}$ equals her initial wealth $\tilde{w}$, and thus does not depend on the interest rate r.

The efficient quality threshold θ^* depends on the interest rate r only directly, and thus the long-run effect is equal to the short-run effect. The quality threshold applied by the bank, $\hat{\theta}(r, R^{SB}(r))$, on the other hand, is not only directly a function of the interest rate r but also indirectly via the repayment $R^{SB}(r)$. The total change of this threshold is

$$\frac{d\hat{\theta}}{dr} = \frac{\partial\hat{\theta}}{\partial r} + \frac{\partial\hat{\theta}}{\partial R^{SB}} \frac{dR^{SB}}{dr}. \tag{28}$$

We know that $\partial\hat{\theta}/\partial r > 0$ and that $\partial\hat{\theta}/\partial R^{SB} < 0$. Thus, if the repayment R^{SB} is increasing in the interest rate, the long-run effect of an interest rate change on the likelihood of zombie lending is weaker than the short-run effect. An interest rate change affects the considerations of all three agents, the entrepreneur, the bank, and the investors. An increase in the interest rate makes the entrepreneur less patient, and thus selling the project at $t = 0$ to investors becomes more attractive. Therefore, to make the entrepreneur accept the bank loan, the repayment needs to be lower. On the other hand, an increase in the interest rate decreases the expected net present value of the project, and thus reduces investors' willingness to pay at $t = 0$. This allows the bank to demand a higher repayment. Finally, for a higher interest rate, the bank has the incentive to liquidate more projects at $t = 1$. The higher interest rate not only decreases the probability of the

[18] Our results apply to empirically documented zombie firms that borrow from banks at a fixed interest rate (fixed repayment R). Göbel and Tavares (2022) find that zombie firms – compared to their non-zombie counterparts – rely more heavily on bank loans with fixed interest rates than on revolving loan facilities that typically have interest rates of a variable line.

entrepreneur profitably selling the project at $t = 2$ but also, in case of a sale, leads to a higher project price P_2. A sufficient (but not necessary) condition for $dR^{SB}/dr > 0$ is that a rise in the interest rate r increases – ceteris paribus – the advantage of bank finance over market finance.[19] In other words, the possibility of early liquidation is particularly valuable if interest rates are high. To obtain an unambiguous result, we, therefore, impose the following simple sufficient condition:

Assumption 3. *The quality of a project is non-negative, i.e.,* $\underline{\theta} \geq 0$.

According to Assumption 3, no project in itself makes negative returns. Note, however, that $\underline{\theta} \geq 0$ does not exclude projects having a negative net present value at $t = 0$ nor liquidation being the efficient decision at $t = 1$. We can then make the following proposition.

Proposition 5. *Suppose that Assumption 3 holds and that* $P_0 = [1+(1+r)\tilde{\gamma}]\mu-(1+r)^2\tilde{I} > 0$. *Then,*

(i) the repayment of the second-best contract R^{SB} *is strictly increasing in the interest rate* r*;*

(ii) under the second-best loan contract, the probability of zombie lending is strictly increasing in the interest rate; i.e.,

$$Z(r) = \int_{\hat{\theta}(r,R^{SB}(r))}^{\theta^*(r)} f(\theta)\,\mathrm{d}\theta \tag{29}$$

is strictly increasing in r.

According to Proposition 5, an anticipated drop (rise) in the interest rate decreases (increases) the probability of zombie lending. As the proof reveals, the bank's quality threshold $\hat{\theta}$ is decreasing in the interest rate. Thus, apparent from (28), the indirect effect of contract adaption on the bank's quality threshold must outweigh the direct effect. While this result may be surprising at first, the rough intuition of the finding can be argued as follows: An increase in the interest rate makes risk-neutral investors less willing to pay for the entrepreneur's project at date $t = 0$, and thus P_0 becomes smaller. In

[19] The expected advantage of bank finance over market finance in terms of $t = 1$ values is

$$\psi(r,\hat{\theta}) = F(\hat{\theta})\tilde{L} + \left(\tilde{\gamma} + \frac{1}{1+r}\right)\left[\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - \mu\right].$$

Note that $\partial\psi/\partial r > 0$ if and only if $\int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta > 0$.

return, the bank adapts the loan contract by demanding a higher repayment R^{SB} from the entrepreneur (participation constraint) ex ante. This higher repayment ultimately leads to a higher incentive for the bank to continue projects at date $t = 1$, and thus zombie lending increases. We investigate the channels behind this finding in more detail in Section 5.1, where we allow for different interest rates for the three types of agents.

In summary, we find that a mere drop in interest rate does not cause long-run zombification, but in fact has a diminishing effect. Translating our result to the real world, low interest rate environments may lead to increased zombie lending within relationship banking in the short-run but not in the long-run. In other words, if interest rates are low in a monetary area for a prolonged period, the economy is not at risk of being crowded by zombie firms. Evidence in line with our result is, for instance, Beer *et al.* (2021) who analyze zombie shares in Austria. They report an especially pronounced decline in the zombie share in the years 2015 until 2017. On a similar note, Banerjee and Hofmann (2021) report weakly decreasing zombie shares post the year 2010 for Japan, Denmark and Germany.[20] Recall that our theory of relationship banking fits best to bank-oriented economies such as Germany and Japan rather than market-oriented economies such as the US or UK.

5. Extensions and Further Implications

5.1. Diverging Time Preferences

In this section, to gain a better understanding of the main drivers behind Proposition 5, we allow for different interest rates across the three types of agents. These diverging interest rates may reflect different time preferences, different opportunity costs, or different alternative investment opportunities. The interest rate of agent $i \in \{B, E, M\}$ is r_i, where subscript B denotes the bank, subscript E the entrepreneur, and subscript M the agents acting in the financial market (the investors).[21] We investigate how a change in the interest rate r_i applied by agent i affects the second-best repayment $R^{SB} = R^{SB}(r_B, r_M, r_E)$ and

[20] The findings regarding Germany are also observed in the data by Blažková and Chmelíková (2022, p. 8), who report "for Germany, the share of zombies has increased during the crisis, but after 2009 it has been gradually declining."

[21] If the entrepreneur chooses market finance at $t = 0$, selling the whole project is only optimal if $r_E \geq r_M$, i.e., if the entrepreneur is less patient, and thus discounts future profits stronger than investors. To keep the analysis as close as possible to the previous analysis, we assume that this is the case.

the quality threshold

$$\hat{\theta}(r_B, R^{SB}) = \frac{(1+r_B)\tilde{L} - R^{SB}}{(1+r_B)\tilde{\gamma}} \tag{30}$$

applied by the bank.[22]

The second-best repayment R^{SB} makes the entrepreneur indifferent between bank finance and her best alternative (market finance or outside option). Hence, it solves

$$\frac{1}{(1+r_E)^2}\left\{\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\hat{\theta})]\right\} - \tilde{w} = \max\left\{\frac{\mu}{(1+r_M)^2} + \frac{\tilde{\gamma}\mu}{1+r_M} - \tilde{I},\ 0\right\} \tag{31}$$

where $\hat{\theta}(r_B, R^{SB})$ is given by (30). The interest rate of investors, r_M, influences the repayment, and thus the threshold $\hat{\theta}$ only if market finance is better than the outside option, i.e., if $P_0 > 0$. Therefore, in the following, we focus on the case $P_0 > 0$.

Proposition 6. *Suppose that $P_0 = [1 + (1+r_M)\tilde{\gamma}]\mu - (1+r_M)^2\tilde{I} > 0$. Then,*

(i) the repayment R^{SB} is strictly increasing and the bank's quality threshold $\hat{\theta}$ is strictly decreasing in the interest rate of investors (market participants): $\partial R^{SB}/\partial r_M > 0$ and $\partial\hat{\theta}/\partial r_M < 0$;

(ii) the repayment R^{SB} is strictly decreasing and the bank's quality threshold $\hat{\theta}$ is strictly increasing in the entrepreneur's interest rate: $\partial R^{SB}/\partial r_E < 0$ and $\partial\hat{\theta}/\partial r_E > 0$;

(iii) the repayment R^{SB} and the bank's quality threshold $\hat{\theta}$ are both strictly increasing in the bank's interest rate: $\partial R^{SB}/\partial r_B > 0$ and $d\hat{\theta}/dr_B > 0$.

If the interest rate of investors r_M increases, then purchasing the project at $t = 0$ becomes less attractive to investors. The entrepreneur's best alternative – market finance – becomes less attractive, and thus the bank can demand a higher repayment. The higher repayment directly translates into a lower quality threshold $\hat{\theta}$.

If, on the other hand, the interest rate of the entrepreneur r_E increases, she discounts future profits more heavily, and thus selling the project to investors at $t = 0$ instead of at $t = 2$ (after intermediate run bank finance) becomes more attractive. This implies that the bank is forced to reduce the repayment, which increases its quality threshold.

Finally, the effect of an increase in the bank's interest rate r_B has a more nuanced effect. If the bank discounts future profits stronger, it has the incentive to terminate more

[22] In this section, we do not investigate how the probability of zombie lending is affected by changes in interest rates. The reason is that for $r_E \neq r_B$ it is not clear how to define the efficient threshold θ^*, and thus zombie lending.

projects. Thus, the direct effect of an increase in r_B on the quality threshold $\hat{\theta}$ is positive. A change in the bank's interest rate also affects the second-best repayment. First, the higher quality threshold implies that – ex ante – the project is less likely to be sold at $t = 2$. Second, a project sold at $t = 2$ obtains a higher price P_2 because an increase in $\hat{\theta}$ increases the average quality of continued projects. In the second-best contract, the repayment is too high from a welfare perspective ($R > \hat{\theta}$), implying that the price effect dominates the probability effect. This allows the bank to demand a higher repayment R^{SB}. The effect of an increase of the bank's interest rate on the quality threshold via the repayment is only of second order such that the threshold is strictly increasing in r_B.

According to Proposition 5 – all agents use an identical interest rate $r = r_B = r_E = r_M$ – an increase in the interest rate decreases the bank's quality threshold. Proposition 6 illustrates that the aforementioned comparative static is driven by two effects. First, an increase in the identical interest rate r increases investors' discounting, which leads to an increase in the repayment and thus a decrease in the quality threshold. Moreover, an increase in the bank's interest rate increases the repayment R^{SB}, which – ceteris paribus – leads to a decrease in the quality threshold. For identical interest rates, these two effects dominate.

5.2. Alternative Investment Opportunities by Investors

In the previous section, we learned that one main driver behind Proposition 5 is that a reduction in the interest rate makes it more attractive for investors to finance the project at the initial date $t = 0$. This effect can be described as a competition effect: the lower the interest rate, the stronger the competition between investors and the bank of being selected as the financial backer for the entrepreneur's project. Due to this effect, a lower interest rate decreases the repayment under the second-best contract and increases the bank's quality threshold. In the long-run, this makes zombie lending less likely for low interest rates.

A reduction in the interest rate may, however, positively affect the return on alternative investments that are available to the investors. For instance, the reduction in interest rates may cause an increase in the demand for corporate stocks, leading to higher expected returns from investing in stocks.[23] Moreover, capital intensive industries benefit from low

[23] Daniel *et al.* (2021) report that low interest rates drive up demand and prices for high-dividend stocks and high-yield bonds. Somewhat related, Domian *et al.* (1996) find that drops in interest rates are followed by excessive stock returns. A theoretical mechanism of how lower nominal interest rates that make liquidity cheaper translate into higher asset prices and investments is proposed by Drechsler

interest rates, and thus can generate higher revenues. In the following, we augment our baseline model by incorporating the latter channel.

A central bank determines the basis interest rate r^*. For simplicity, we assume that the relationship bank uses this basis interest rate, i.e., $r_B = r^*$. The interest rate applied by the entrepreneur, r_E, reflects her idiosyncratic time preference and is independent of r^*. The interest rate used by investors r_M is the net return they can achieve from alternative investments.

There is a large number of homogeneous firms that operate each with a fixed amount of equity k_E.[24] Each firm chooses an amount of outside capital k_O. A firm invests in $t = 0$ (and in $t = 1$) and generates a gross return of $B(k_E + k_O)$ in $t + 1$, with $B'(\cdot) > 0$ and $B''(\cdot) < 0$. A firm's profit (net present value) is $\pi(r^*) = B(k_E + k_O^*) - (1 + r^*)k_O^*$, where $k_O^*(r^*)$ is the profit-maximizing amount of outside capital.[25] Thus, the net return on equity is

$$r_M(r^*) = \frac{\pi(r^*)}{k_E} - 1. \tag{32}$$

Each investor can decide to finance such a firm instead of the entrepreneur's project. An investor prefers to finance the entrepreneur's project if it has an expected net return that is weakly larger than $r_M(r^*)$.

We focus on situations where market finance is the entrepreneur's best alternative to bank finance, i.e., we assume that

$$P_0 := \frac{\mu}{(1 + r_M)^2} + \frac{\tilde{\gamma}\mu}{1 + r_M} - \tilde{I} > 0. \tag{33}$$

We can now state the following result.

Proposition 7. *Suppose that $P_0 > 0$. An increase in the basis interest rate r^**

(i) decreases the net return investors demand from the entrepreneur, $dr_M/dr^* = -k_O^*/k_E < 0$;

(ii) increases the quality threshold $\hat{\theta}(R^{SB})$ that the bank applies under the second-best contract, $d\hat{\theta}/dr^* > 0$.

Moreover, the bank's quality threshold $\hat{\theta}(R^{SB})$ reacts stronger to a change in the basis interest rate r^*, the stronger the net return r_M reacts, i.e., the larger $|dr_M/dr^*|$ is.

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et al. (2018).

[24] Assuming a fixed amount of equity has the advantage that profit-maximization is equivalent to maximizing the rate of return on equity.

[25] We assume that k_O^* is determined by the first-order condition of profit maximization. Imposing the Inada conditions $\lim_{k\to 0} B'(k_E + k) = \infty$ and $\lim_{k\to\infty} B'(k_E + k) = 0$ is sufficient.

If the central bank interest rate r^* increases, the productivity of firms declines, which in turn reduces the return on equity, part (i) of Proposition 7. An increase in the interest rate r^* has two effects on the bank's quality threshold $\hat{\theta}$. First, there is the direct positive effect on $\hat{\theta}$: If the interest rate is higher, the bank has the incentive to liquidate more often. Second, a change in the basis interest rate changes the second-best repayment R^{SB}. Regarding the repayment, there are two opposing effects. On the one hand, the bank liquidates more often, which increases the second-period price P_2. This allows the bank to demand a higher repayment. On the other hand, if the interest rate r^* increases, financing the entrepreneur rather than one of the homogeneous firms becomes more attractive for investors. This forces the bank to reduce the repayment. The former effect dominates if $|dr_M/dr^*| \approx 0$, while the latter dominates if $|dr_M/dr^*|$ is large. In any case, the overall effect on the quality thresholds is unambiguous: a higher interest rate r^* increases the bank's quality threshold.

Proposition 7 alludes to the concern that a low basis interest rate may lead to more zombie lending not only in the short-run but also in the long-run. This concern can be mitigated by strict financial regulations, e.g. capital requirements. A higher required share of equity to outside capital reduces the leverage of the publicly traded companies, and thus their return on equity. To see this mathematically, note that $|dr_M/dr^*| = k_O^*/k_E$ is strictly decreasing in k_E.

5.3. Bank's Capital Structure

Empirical evidence suggests that zombie lending is a more pronounced problem if the lender (the bank) is itself in a weak financial position (Peek and Rosengren, 2005; Acharya *et al.*, 2021b; Blattner *et al.*, forthcoming). In other words, a bank with lower equity to outside capital ratio has a stronger incentive to roll over loans of poor quality. In the following, we consider a simple extension of the baseline model.

To address the issue of bank capital structure, we now assume that the bank finances the investment partially with equity and partially with outside finance. More precisely, share $\alpha \in (0, 1]$ of the investment $\tilde{I} - \tilde{d}$ is financed by bank equity and share $1 - \alpha$ by deposits. The bank pays an interest $r_D < r$ on deposits. To rule out trivial cases, we assume that the bank can repay the deposits also in case of project liquidation. Moreover, we focus on the second-best loan contract with $\tilde{d}^{SB} = \tilde{w}$. Under the second-best contract, the repayment $R^{SB} = R$ is determined by the entrepreneur's participation constraint, and thus is independent of the bank's capital structure. The bank keeps the deposits on the balance sheet for two periods if the entrepreneur's loan is continued at $t = 1$ but only for one period if the loan is terminated at $t = 1$.

The bank prefers to roll over the entrepreneur's loan at $t = 1$ if and only if

$$\tilde{\gamma}\theta + \frac{R^{SB}}{1+r} - (1-\alpha)\frac{(1+r_D)^2(\tilde{I}-\tilde{w})}{1+r} \geq L - (1-\alpha)(1+r_D)(\tilde{I}-\tilde{w}). \qquad (34)$$

The difference between (34) and the respective condition in the baseline model is that the bank needs to repay the deposits $(\tilde{I} - \tilde{w})$ plus interest payments. The next result is readily obtained from (34).

Proposition 8. *Suppose the bank's equity share is α and it pays an interest $r_D < r$ on deposits. Then, the bank's quality threshold is higher, the higher the equity share: $\partial\hat{\theta}/\partial\alpha > 0$.*

The lower a bank's quality threshold $\hat{\theta}$, the higher is the scope for zombie lending – i.e., roll-over of loans from projects with inefficiently low returns. Thus, according to Proposition 8, weakly capitalized or even under-capitalized banks are particularly likely to engage in zombification.

5.4. Booms and Busts

Zombification seems to be particularly pronounced during economic downturns. Banerjee and Hofmann (2021) and De Martiis and Peter (2021) report that the share of zombie firms rises during recessions. For instance, De Martiis and Peter (2021) analyze the share of zombie firms in eight European countries from 1990 till 2018. For this period, they investigate how three recession events, the Dot-com Bubble, the GFC, and the European Debt Crisis, affected the likelihood of zombie lending. They point out that recession events are likely to be a primary cause for firms to become over-indebted. The recession alone, however, can hardly explain why these non-viable firms stay alive as they do according to the data of De Martiis and Peter (2021).

In the following, we investigate how an (unexpected) change in the economic conditions at the beginning of $t = 1$ – i.e., for given contracts – affects the probability of zombie lending. If there is an economic downturn at the beginning of $t = 1$, this affects the prospects regarding the project's returns in $t = 1$ and likely also in $t = 2$. Moreover, in an economic downturn, prices may drop, affecting the value of the entrepreneur's assets, e.g. the collateral and the value of the company's physical capital. In other words, the liquidation value of the project is reduced in an economic downturn. We model this by assuming that the project's quality is $\alpha\theta$ and the liquidation value is $\alpha\tilde{L}$, with $\alpha > 0$. For $\alpha < 1$ the economy is in a recession and for $\alpha > 1$ in a boom. We focus on a given second-best contract (d^{SB}, R^{SB}), where R^{SB} is optimal for the neutral economic condition $\alpha = 1$. We restrict the attention to drops in values that are not too severe, i.e., we assume that α is sufficiently large so that $P_2 = \mathbb{E}[\alpha\theta|\theta \geq \hat{\theta}(\alpha)] > R^{SB}$. The price that the entrepreneur obtains at $t = 2$ is larger than the repayment, and thus the bank always obtains R^{SB} in $t = 2$.

First, note that the efficient quality threshold θ^* is independent of α because all relevant payments from $t = 1$ onward – both the project revenues and the liquidation value – are scaled by α. The bank, however, prefers to roll over the loan if and only if

$$\tilde{\gamma}\alpha\theta + \frac{R^{SB}}{1+r} \geq \alpha\tilde{L}. \tag{35}$$

The roll-over decision of the bank hinges on the economic state α because the repayment is fixed ex ante and does not depend on the economic situation.

Proposition 9. *The probability of zombie lending $Z(\alpha) = \int_{\hat{\theta}(\alpha)}^{\bar{\theta}} f(\theta)\,\mathrm{d}\theta$ increases (decreases) in a recession (boom), i.e., $dZ/d\alpha < 0$.*

According to Proposition 9 and in line with empirical evidence, zombie lending increases if the economy turns into a recession. With the repayment being fixed ex ante, the bank has the incentive to continue the project for more quality levels if the liquidation value and the project's returns decrease. Intuitively, the relationship bank prefers to 'speculate' on obtaining the (ex ante) contracted repayment in the future rather than realizing the busted liquidation value.

6. Robustness and Discussion

6.1. Alternative Contracts: Repayments in $t = 1$ and $t = 2$

Suppose that at $t = 0$, the bank offers a contract $\mathcal{C} = (\tilde{d}, \tilde{R}_1, R_2)$ that specifies (i) the own contribution of the entrepreneur to the investment $\tilde{d} \leq \tilde{w}$, (ii) a repayment $\tilde{R}_1$ to be made at the end of $t = 1$, and (iii) a repayment R_2 to be made at $t = 2$. The entrepreneur keeps the control and cash-flow rights at $t = 1$. If, however, the bank learns at the beginning of $t = 1$ that the entrepreneur will be unable to make the repayment $\tilde{R}_1$, it can force the illiquid entrepreneur to liquidate her business. The bank can also decide to roll over the loan even though the entrepreneur is not able to pay the full obligation $\tilde{R}_1$.

To simplify the exposition, we focus on the case $\tilde{d} = \tilde{w}$. Moreover, by the argument outlined for the baseline model, we know that $R_2 \leq P_2 = \mathbb{E}[\theta|\theta \geq \hat{\theta}(R_2)]$. If $\tilde{\gamma}\theta < \tilde{R}_1$, and thus the entrepreneur is insolvent, the bank prefers the continuation if and only if

$$\tilde{\gamma}\theta + \frac{R_2}{1+r} \geq \min\{\tilde{L}, \tilde{R}_1\} \iff \theta \geq \frac{(1+r)\min\{\tilde{L}, \tilde{R}_1\} - R_2}{\tilde{\gamma}(1+r)} =: \hat{\theta}. \tag{36}$$

For $\tilde{R}_1 \geq \tilde{L}$ and $R_2 = \theta^*$ we have $\hat{\theta} = \theta^*$, i.e., the first-best quality threshold is implemented.

If the bank can extract larger rents from the entrepreneur, it can increase its profit by either increasing $\tilde{R}_1$ or R_2. Increasing $\tilde{R}_1 \geq \tilde{L}$ does not distort the roll-over decision but increases the bank's expected total repayment. Once $\tilde{R}_1 = \tilde{\gamma}\bar{\theta}$, a further increase of $\tilde{R}_1$ does not increase the bank's expected profit. If this is the case, the bank has the incentive to demand a repayment $R_2 > \theta^*$. Now, the contract $\mathcal{C} = (\tilde{d} = \tilde{w}, \tilde{R}_1 = \tilde{\gamma}\bar{\theta}, R_2)$ is equivalent to the second-best contract analyzed in the baseline model. In other words, if the entrepreneur is wealth constrained, $\tilde{w} < \tilde{d}^*$, the loan contract $\mathcal{C} = (\tilde{d}, \tilde{R}_1, R_2)$ specifies a first-period repayment $\tilde{R}_1$ so that the effective continuation-liquidation decision at $t = 1$ is made by the bank.

In practice there can be several reasons why the signed contract leaves a rent to the entrepreneur at $t = 1$, i.e., $\tilde{R}_1 < \tilde{\gamma}\bar{\theta}$ is optimal. One reason could be a non-contractible effort by the entrepreneur that is important for project success. Our simple model abstracts from any moral hazard issues. Note, however, the tighter the constraint on $\tilde{R}_1$ from above (e.g. due to moral hazard issues), the higher the initial investment, $\tilde{I} - \tilde{d}$, or the repayment R_2 that the bank demands. On this account, a further constraint on $\tilde{R}_1$ implies even more scope for zombie lending.

6.2. Bank Competition

Throughout the paper, we assumed that a monopolistic bank learns the quality of the project at an intermediate date and makes a take-it-or-leave-it contract offer. The bank's offer is constrained by the risk-neutral investors' offer to the entrepreneur at date $t = 0$. In the baseline model, however, no other bank can monitor the project and is willing to finance it. The terms of the second-best contract, under which zombie lending occurs, are determined by the entrepreneur's participation constraint. In the following, we show that zombie lending can also occur under bank competition where the entrepreneur's participation constraint does not determine the equilibrium repayment.

Suppose that there are several – at least two – banks that can create a relationship with the entrepreneur. These banks, who are all identical, compete at date $t = 0$ à la Bertrand by making a loan contract offer (d, R). To simplify the exposition we assume a cashless entrepreneur, i.e., $w = 0$, and that there is no financial market. Note that if bank finance occurs in equilibrium, the next best alternative for the entrepreneur is to take up a loan from another bank. Thus, we can abstract from market finance without loss in generality.

Furthermore, we assume that a bank that fully finances the project ($d = 0$) and charges the highest feasible repayment $\bar{R}$ makes a strictly positive expected profit.

Assumption 4. $\pi_B(0, \bar{R}) > 0$.

The assumption implies that the expected surplus generated by efficient bank finance ($R = \theta^*$) is strictly positive. To be able to state a concise result, we define the following threshold

$$\bar{d} := -F(\theta^*)L - \gamma \int_{\theta^*}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\theta^*)]\theta^* + c + I.$$

Finally, we focus on symmetric equilibria of the bank competition game.

Proposition 10. *Suppose that Assumption 4 holds. If $\bar{d} > 0$, the equilibrium loan contract (d^C, R^C) under bank competition specifies $d^C = 0$ and $R^C \in (R^*, \bar{R})$ so that $\pi_B(0, R^C) = 0$.*

According to Proposition 10, if the expected bank profit from the contract that induces efficient continuation ($R = \theta^*$) is negative, the equilibrium contract specifies an inefficiently high repayment $R^C > R^* = \theta^*$. Thus, under the competitive loan contract (d^C, R^C), zombie lending takes place for projects of quality $\theta \in [\hat{\theta}(R^C), \theta^*)$. Moreover, as a bank's continuation decision for a given contract is independent of the degree of bank competition, Proposition 4 still applies. In other words, if the entrepreneur signed an equilibrium loan contract under bank competition (d^C, R^C), an unanticipated drop in the interest rate increases the probability of zombie lending.

What is the intuition behind Proposition 10? The equilibrium loan contract maximizes the expected profit of the entrepreneur subject to a bank's break-even constraint, the entrepreneur's limited funds (LL), and taking the roll-over decision (RD) into account. If (LL) is slack, the optimal contract maximizes the joint surplus, and thus specifies $R = R^*$ so that the roll-over decision is efficient. If (LL) is binding, and thus $\bar{d} > 0$, the contract specifies $d^C = 0$. In this case, the bank can break even only for repayments higher than $R^* = \theta^*$. As a result, the equilibrium contract specifies a repayment $R^C > R^*$ and zombie lending occurs for qualities $\theta \in [\hat{\theta}(R^C), \theta^*)$. Recall that $\hat{\theta}(R^C) < \hat{\theta}(R^*) = \theta^*$.

7. Conclusion

In this paper, we analyze a simple zombie lending mechanism. A relationship bank may grant a loan to an entrepreneur who possesses a project of ex ante unknown quality. We show that within a second-best contract – that arises in equilibrium if the entrepreneur is cash constrained – the relationship bank continues projects of inefficiently low qualities: zombie lending occurs. The reason is that the binding upper bound on the entrepreneur's initial outlay directly translates into an inefficiently high ex post repayment demanded by the relationship bank. The latter fact, in turn, leads to a distorted continuation decision.

Investigating the bank's motive for inefficient roll-over decisions further, we introduce interest rate shocks. In case the interest rate drops unexpectedly, i.e., the bank faces a 'new' continuation decision for a predetermined second-best contract, the probability of zombie lending increases. Intuitively, the bank becomes more patient when the interest rate drops, and hence continuing the project and receiving the inefficiently high ex post repayment becomes more attractive. Interestingly, we find that the relationship between the bank's zombie lending behavior and the interest rate is inverted in the long run, i.e., where contracts are adapted. In other words, the probability of zombie lending decreases with lower interest rates. Since lower interest rates increase the market investors' willingness to pay for the entrepreneur's project, the relationship bank reacts by offering a contract with a lower ex post repayment. As a consequence, the bank's roll-over decision becomes more efficient, i.e., the bank continues fewer zombie projects. In an extension, we show that this effect mitigates if a low interest rate, say a low basic interest rate of the central bank, increases the attractiveness of alternative investment opportunities that market investors have.

A. Mathematical Appendix

Proof of Observation 1. The result follows readily from comparing the expected surplus of market finance (2), the expected surplus from efficient bank finance (4), and the surplus from no finance, which is zero. □

Proof of Proposition 1. For $R = R^*$, we have $\hat{\theta}(R) = \theta^*$ and $P_2 = \mathbb{E}[\theta \mid \theta \geq \theta^*]$. This implies that for repayment R^* the entrepreneur is indifferent between accepting the bank loan (d, R^*) and her next best alternative if and only if

$$d = [1 - F(\theta^*)]\big\{\mathbb{E}[\theta \mid \theta \geq \theta^*]\big\} - \max\{(1+\gamma)\mu - I, 0\} \tag{A.1}$$

$$= \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\,\mathrm{d}\theta - \max\{(1+\gamma)\mu - I, 0\}. \tag{A.2}$$

Note that $P_0 = (1+\gamma)\mu - I$. If bank finance is efficient and all the additional surplus from bank finance is extracted by the bank – i.e., participation is binding – then offering a loan contract that implements efficient continuation maximizes the bank's profits. □

Proof of Proposition 2. The bank maximizes its profit subject to the entrepreneur's participation constraint, $\pi_E(d, R) \geq \max\{P_0, 0\}$, and the limited liability constraint, $d \leq w$. The first-best contract (d^*, R^*) satisfies the participation but violates the limited liability constraint, $w < d^*$. With d being an ex ante one-to-one transfer between the entrepreneur and the bank, the second-best optimal amount financed by the entrepreneur is $d^{SB} = w$.

The expected profit of the bank is

$$\pi_B(d^{SB}, R) = F(\hat{\theta}(R))[L - c - I + w] + [1 - F(\hat{\theta}(R))]\big\{\gamma\mathbb{E}[\theta | \theta \geq \hat{\theta}(R)] + R - c - I + w\big\}. \tag{A.3}$$

Simplifying the above expression yields

$$\pi_B(d^{SB}, R) = F(\hat{\theta}(R))L + \gamma \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta + [1 - F(\hat{\theta}(R))]R - (c + I - w). \tag{A.4}$$

Taking the derivative of π_B with respect to the repayment R yields

$$\frac{\partial \pi_B}{\partial R} = f(\hat{\theta})\frac{d\hat{\theta}}{dR}L - \gamma\hat{\theta} f(\hat{\theta})\frac{d\hat{\theta}}{dR} + [1 - F(\hat{\theta})] - f(\hat{\theta})\frac{d\hat{\theta}}{dR}R$$

$$= -f(\hat{\theta})\frac{1}{\gamma}\underbrace{[L - \gamma\hat{\theta} - R]}_{=0} + 1 - F(\hat{\theta}) > 0 \tag{A.5}$$

The term in square brackets equals zero by the definition of $\hat{\theta}$. Thus, the bank strictly prefers a higher repayment R.

The expected profit of the entrepreneur is

$$\pi_E(d^{SB}, R) = F(\hat{\theta}(R))(-w) + [1 - F(\hat{\theta}(R))]\{\mathbb{E}[\theta|\theta \geq \hat{\theta}(R)] - R - w\}$$
$$= \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\hat{\theta}(R))]R - w. \tag{A.6}$$

Note that $\pi_E(d^{SB}, R^*) > \max\{P_0, 0\}$ because $\pi_E(d^*, R^*) = \max\{P_0, 0\}$ and $d^* > w = d^{SB}$. Moreover, $\pi_E(d^{SB}, \bar{R}) = -w$, which implies that for $R > \bar{R}$ the participation constraint is violated. Recall that $\bar{R}$ is implicitly defined by $\mathbb{E}[\theta|\theta \geq \hat{\theta}(\bar{R})] = \bar{R}$. Hence, $R^{SB} \in (R^*, \bar{R}]$.

Taking the partial derivative of the entrepreneur's expected profit with respect to R yields

$$\frac{\partial \pi_E}{\partial R} = -\hat{\theta} f(\hat{\theta}) \frac{d\hat{\theta}}{dR} - [1 - F(\hat{\theta})] + R f(\hat{\theta}) \frac{d\hat{\theta}}{dR}$$
$$= -[R - \hat{\theta}] f(\hat{\theta}) \frac{1}{\gamma} - [1 - F(\hat{\theta})]. \tag{A.7}$$

For $R > R^*$ we have $\hat{\theta}(R) < \theta^*$ and, thus, $\partial \pi_E / \partial R < 0$.

The bank's expected profit is strictly increasing in R and the entrepreneur's expected profit is strictly decreasing in R. Thus, the second-best optimal repayment R^{SB} solves $\pi_E(d^{SB}, R) = \max\{P_0, 0\}$. □

Proof of Corollary 1. The finding follows directly from the observation that $R^{SB} > R^*$ for $w < d^*$. □

Proof of Proposition 3. The first-best outcome is described in Observation 1. If a project is not financed in the first-best, it is also not financed in equilibrium. Moreover, if market finance is efficient, it also occurs in equilibrium because the full surplus of this channel accrues to the entrepreneur. Similarly, if bank finance is efficient and the first-best loan contract is offered by the bank ($w \leq d^*$), then bank finance occurs in equilibrium. The remaining question is, when is the second-best loan contract (d^{SB}, R^{SB}) offered in equilibrium. The bank's offer just compensates the entrepreneur for her best alternative option. Thus, the second-best loan contract is offered as long as the resulting expected bank profits are non-negative. This is the case if and only if $c \leq \bar{c}^{SB}$, which is characterized by (19).

Differentiation of (19) with respect to I yields

$$\frac{d\bar{c}^{SB}}{dI} = \frac{dR^{SB}}{dI} \left\{ 1 - F(\hat{\theta}) - \frac{d\hat{\theta}}{dR} f(\hat{\theta}) [\underbrace{\gamma\hat{\theta} + R^{SB} - L}_{=0}] \right\} - 1 \tag{A.8}$$
$$= \frac{dR^{SB}}{dI} [1 - F(\hat{\theta})] - 1. \tag{A.9}$$

The second-best repayment $R^{SB}(I)$ is implicitly defined by

$$\int_{\hat{\theta}(R^{SB})}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\hat{\theta}(R^{SB}))] - w = \max\{(1+\gamma)\mu - I, 0\}. \tag{A.10}$$

First, suppose that $(1+\gamma)\mu - I \geq 0$. The implicit differentiation of (A.10) with respect to I yields

$$-\hat{\theta} f(\hat{\theta}) \frac{d\hat{\theta}}{dR}\frac{dR^{SB}}{dI} + R^{SB} f(\hat{\theta})\frac{d\hat{\theta}}{dR}\frac{dR^{SB}}{dI} - [1 - F(\hat{\theta})]\frac{dR^{SB}}{dI} = -1. \tag{A.11}$$

Rearranging the above expression and using the fact that $d\hat{\theta}/dR = -\gamma^{-1}$ yields

$$\frac{dR^{SB}}{dI} = \frac{1}{1 - F(\hat{\theta}) + \frac{1}{\gamma}(R^{SB} - \hat{\theta}) f(\hat{\theta})} > 0. \tag{A.12}$$

□

Inserting (A.12) in (A.9) yields

$$\frac{d\bar{c}^{SB}}{dI} = -\frac{\frac{1}{\gamma}(R^{SB} - \hat{\theta}) f(\hat{\theta})}{1 - F(\hat{\theta}) + \frac{1}{\gamma}(R^{SB} - \hat{\theta}) f(\hat{\theta})} \in (-1, 0). \tag{A.13}$$

Recall that $R^{SB} > R^* = \theta^* > \hat{\theta}(R^{SB})$.

Second, suppose that $I > (1+\gamma)\mu$. In this case, R^{SB} is independent of I, which is apparent from (A.10). Thus, $dR^{SB}/dI = 0$. Now, using (A.9), we immediately obtain that

$$\frac{d\bar{c}^{SB}}{dI} = -1. \tag{A.14}$$

This concludes the proof.

Proof of Corollary 2. The finding follows directly from the proof of Proposition 3 in combination with Corollary 1. □

Proof of Proposition 4. Taking the derivative of $Z(r)$ with respect to r – for a constant repayment R^{SB} – yields

$$Z'(r) = f(\theta^*)\frac{d\theta^*}{dr} - f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r}. \tag{A.15}$$

To sign the above expression, we first need to determine $d\theta^*/dr$ and $\partial\hat{\theta}/dr$. Taking the partial derivative of (24) with respect to r yields

$$\begin{aligned}\frac{\partial\hat{\theta}}{\partial r} &= \frac{\tilde{L}(1+r)\tilde{\gamma} - \tilde{\gamma}[(1+r)\tilde{L} - R^{SB}]}{\tilde{\gamma}(1+r)^2} \\ &= \frac{R^{SB}}{\tilde{\gamma}(1+r)^2} > 0. \end{aligned}\tag{A.16}$$

Taking the partial derivative of (22) with respect to r yields

$$\frac{d\theta^*}{dr} = \frac{\tilde{L}[1+(1+r)\tilde{\gamma}] - \tilde{\gamma}(1+r)\tilde{L}}{[1+(1+r)\tilde{\gamma}]^2} = \frac{\tilde{L}}{[1+(1+r)\tilde{\gamma}]^2} > 0. \tag{A.17}$$

Using the definition of θ^* allows us to write the above derivative as

$$\frac{d\theta^*}{dr} = \frac{\theta^*}{(1+r)[1+(1+r)\tilde{\gamma}]}. \tag{A.18}$$

By Assumption 2 it holds that $f(\theta^*) \leq f(\hat{\theta})$. Thus, $Z'(r) \leq f(\hat{\theta})[d\theta^*/dr - \partial\hat{\theta}/\partial r]$, which implies that $Z'(r) < 0$ for $\partial\hat{\theta}/\partial r > d\theta^*/dr$. Note that $\partial\hat{\theta}/\partial r > d\theta^*/dr$ is equivalent to

$$R^{SB}(1+r)[1+\tilde{\gamma}(1+r)] > \theta^*\tilde{\gamma}(1+r)^2 \tag{A.19}$$

$$\iff R^{SB}(1+r) + \tilde{\gamma}(1+r)^2[R^{SB} - \theta^*] > 0. \tag{A.20}$$

The above claim is true because $R^{SB} > \theta^*$ by the assumption that the parties signed the second-best contract. □

Proof of Proposition 5. Under the second-best optimal loan contract, the repayment $R^{SB} \in (R^*, \bar{R})$ solves

$$\frac{1}{(1+r)^2}\left(\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1-F(\hat{\theta})]R^{SB}\right) - \tilde{w} = \frac{\mu}{1+r}\left(\tilde{\gamma} + \frac{1}{1+r}\right) - \tilde{I}, \tag{A.21}$$

where

$$\hat{\theta}(r, R^{SB}(r)) = \frac{(1+r)\tilde{L} - R^{SB}}{(1+r)\tilde{\gamma}}. \tag{A.22}$$

In the above condition determining $R^{SB}(r)$ we use the fact that the entrepreneur's best alternative to bank finance is market finance, i.e., that $P_0 > 0$. The implicit differentiation of (A.21) with respect to r yields

$$\frac{-2}{(1+r)^3}\left\{\int_{\hat{\theta}}^{\bar{\theta}} -[1-F(\hat{\theta})]R^{SB}\right\} + \frac{1}{(1+r)^2}\left\{-\hat{\theta}f(\hat{\theta})\frac{d\hat{\theta}}{dr} + f(\hat{\theta})R^{SB}\frac{d\hat{\theta}}{dr} - [1-F(\hat{\theta})]\frac{dR^{SB}}{dr}\right\} = \frac{-2\mu}{(1+r)^3} - \frac{\gamma\mu}{(1+r)^2}. \tag{A.23}$$

Note that

$$\frac{d\hat{\theta}}{dr} = \frac{1}{(1+r)\tilde{\gamma}}\left[\frac{R^{SB}}{1+r} - \frac{dR^{SB}}{dr}\right]. \tag{A.24}$$

Inserting (A.24) in (A.23) and rearranging yields

$$\frac{dR^{SB}}{dr} = \frac{2\tilde{\gamma}[\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta] + \tilde{\gamma}^2(1+r)\mu}{(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})} + \frac{2(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})}{(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})}\,\frac{R^{SB}}{1+r}. \quad (A.25)$$

By Assumption 3 it holds that $\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta > 0$ and thus $dR^{SB}/dr > 0$.

We proceed by inserting (A.25) into (A.24) and obtain

$$\frac{d\hat{\theta}}{dr} = \frac{-1}{(1+r)\tilde{\gamma}}\left\{\frac{(1+r)\tilde{\gamma}^2\mu + 2\tilde{\gamma}[\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta]}{\tilde{\gamma}(1+r)[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\theta)} + \frac{R^{SB}}{1+r}\,\frac{\tilde{\gamma}(1+r)[1-F(\hat{\theta})]}{\tilde{\gamma}(1+r)[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\theta)}\right\} < 0. \quad (A.26)$$

Finally, recall that $d\theta^*/dr > 0$ and thus $Z(r) = \int_{\hat{\theta}}^{\theta^*} f(\theta)\,\mathrm{d}\theta$ is strictly increasing in r. □

Proof of Proposition 6. The second-best repayment $R^{SB} = R^{SB}(r_B, r_E, r_M)$ solves

$$\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1-F(\hat{\theta})]R^{SB} - (1+r_E)^2\tilde{w} = \left\{[1+(1+r_M)\tilde{\gamma}]\mu - (1+r_M)^2\tilde{I}\right\}\frac{(1+r_E)^2}{(1+r_M)^2}, \quad (A.27)$$

where

$$\hat{\theta}(r_B, R^{SB}) = \frac{\tilde{L}(1+r_B) - R^{SB}}{\tilde{\gamma}(1+r_B)}. \quad (A.28)$$

Note that

$$\frac{\partial\hat{\theta}}{\partial r_i} = \frac{-1}{\tilde{\gamma}(1+r_B)}\,\frac{\partial R^{SB}}{\partial r_i} \quad \text{for } i = E, M. \quad (A.29)$$

First, we investigate the comparative static with respect to r_M. The differentiation of (A.27) with respect to r_M yields

$$-\hat{\theta} f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r_H} + f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r_H}R^{SB} - [1-F(\hat{\theta})]\frac{\partial R^{SB}}{\partial r_H} = (1+r_E)^2\left[\frac{-2\mu}{(1+r_M)^3} + \frac{-\tilde{\gamma}\mu}{(1+r_M)^2}\right]. \quad (A.30)$$

We rearrange the above expression and obtain

$$\frac{\partial R^{SB}}{\partial r_M} = \frac{\tilde{\gamma}(1+r_B)(1+r_E)^2[2+\tilde{\gamma}(1+r_M)]\mu}{(R^{SB}-\hat{\theta})f(\hat{\theta}) + \tilde{\gamma}(1+r_B)[1-F(\hat{\theta})]} > 0. \quad (A.31)$$

From (A.31) together with (A.29) it follows immediately that $\partial\hat{\theta}/\partial r_M < 0$.

Next, we implicitly differentiate (A.27) with respect to r_E and obtain

$$-\hat{\theta}f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r_E} + f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r_E}R^{SB} - [1 - F(\hat{\theta})]\frac{\partial R^{SB}}{\partial r_E} - 2(1 + r_E)\tilde{w} = 2(1 + r_E)\tilde{P}_0, \quad \text{(A.32)}$$

where

$$\tilde{P}_0 = -\tilde{I} + \frac{\tilde{\gamma}\mu}{1 + r_M} + \frac{\mu}{(1 + r_M)^2} > 0 \quad \text{(A.33)}$$

by assumption. We rearrange the above expression and obtain

$$\frac{\partial R^{SB}}{\partial r_E} = -\frac{2\tilde{\gamma}(1 + r_B)(1 + r_E)\tilde{P}_0}{(R^{SB} - \hat{\theta})f(\hat{\theta}) + \tilde{\gamma}(1 + r_B)[1 - F(\hat{\theta})]} < 0. \quad \text{(A.34)}$$

From (A.34) together with (A.29) it follows that $\partial\hat{\theta}/\partial r_E > 0$.

Finally, we investigate the comparative static with respect to r_B. First, note that

$$\frac{d\hat{\theta}}{dr_B} = \frac{R^{SB}}{\tilde{\gamma}(1 + r_B)^2} - \frac{1}{\tilde{\gamma}(1 + r_B)}\frac{\partial R^{SB}}{\partial r_B}. \quad \text{(A.35)}$$

The implicit differentiation of (A.27) with respect to r_B yields

$$-\hat{\theta}f(\hat{\theta})\frac{d\hat{\theta}}{dr_B} + f(\hat{\theta})\frac{d\hat{\theta}}{dr_B}R^{SB} - [1 - F(\hat{\theta})]\frac{\partial R^{SB}}{\partial r_B} = 0. \quad \text{(A.36)}$$

Inserting (A.35) into (A.35) and rearranging yields

$$\frac{\partial R^{SB}}{\partial r_B} = \frac{R^{SB}}{1 + r_B}\frac{(R^{SB} - \hat{\theta})f(\hat{\theta})}{(R^{SB} - \hat{\theta})f(\hat{\theta}) + \tilde{\gamma}(1 + r_B)[1 - F(\hat{\theta})]} > 0. \quad \text{(A.37)}$$

To conclude the proof note that

$$\frac{d\hat{\theta}}{dr_B} = \frac{1}{\tilde{\gamma}(1 + r_B)}\left[\frac{R^{SB}}{1 + r_B} - \frac{\partial R^{SB}}{\partial r_B}\right]. \quad \text{(A.38)}$$

Inserting (A.38) into (A.37) reveals that $d\hat{\theta}/dr_B > 0$. □

Proof of Proposition 7. First, we prove part (i): Note that

$$r_M(r^*) = \frac{B(k_E + k_O^*(r^*)) - (1 + r^*)k_O^*(r^*)}{k_E} - 1. \quad \text{(A.39)}$$

Taking the derivative with respect to r^* yields

$$\begin{aligned}\frac{dr_M}{dr^*} &= \frac{1}{k_E}\left[B'(k_E + k_O^*)\frac{dk_O^*}{dr^*} - (1 + r^*)\frac{dk_O^*}{dr^*} - k_O^*\right] \\ &= -\frac{k_O^*}{k_E} < 0. \end{aligned} \quad \text{(A.40)}$$

Next, we prove part (ii). The second-best repayment $R^{SB} = R^{SB}(r^*)$ makes the entrepreneur indifferent between bank finance and market finance:

$$\frac{1}{(1+r_M(r^*))^2}\left[\int_{\hat{\theta}(r^*)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1-F(\hat{\theta}(r^*))]R^{SB}(r^*)\right] - \tilde{w}$$
$$= \frac{\mu}{(1+r_M(r^*))^2} + \frac{\tilde{\gamma}\mu}{1+r_M(r^*)} - \tilde{I}. \quad \text{(A.41)}$$

Recall that

$$\frac{d\hat{\theta}}{dr^*} = \frac{1}{(1+r^*)\tilde{\gamma}}\left[\frac{R^{SB}}{1+r^*} - \frac{dR^{SB}}{dr^*}\right]. \quad \text{(A.42)}$$

Multiplying both sides of (A.41) with $(1+r_E)^2$ and then implicitly differentiating with respect to r^* yields

$$-\hat{\theta}f(\hat{\theta})\frac{d\hat{\theta}}{dr^*} + f(\hat{\theta})\frac{d\hat{\theta}}{dr^*}R^{SB} - [1-F(\hat{\theta})]\frac{dR^{SB}}{dr^*}$$
$$= (1+r_E)^2\left[\frac{-2\mu}{(1+r_M)^3}\frac{dr_M}{dr^*} - \frac{\tilde{\gamma}\mu}{(1+r_M)^2}\frac{dr_M}{dr^*}\right]. \quad \text{(A.43)}$$

We insert (A.42) into (A.43) and solve for

$$\frac{dR^{SB}}{dr^*} = \frac{(R^{SB}-\hat{\theta})f(\hat{\theta})}{(R^{SB}-\hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]}\frac{R^{SB}}{1+r^*}$$
$$+ \frac{(1+r^*)\tilde{\gamma}(1+r_E)^2\mu[2+\tilde{\gamma}(1+r_M)]}{(1+r_M)^3\{(R^{SB}-\hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]\}}\frac{dr_M}{dr^*}. \quad \text{(A.44)}$$

Inserting (A.44) into (A.42) yields

$$\frac{d\hat{\theta}}{dr^*} = \frac{1}{(1+r^*)\tilde{\gamma}}\left[\frac{(1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]}{(R^{SB}-\hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]}\frac{R^{SB}}{1+r^*}\right.$$
$$\left. + \frac{(1+r^*)\tilde{\gamma}(1+r_E)^2\mu[2+\tilde{\gamma}(1+r_M)]}{(1+r_M)^3\{(R^{SB}-\hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]\}}\frac{dr_M}{dr^*}\right]. \quad \text{(A.45)}$$

The above equation allows us to conclude that $d\hat{\theta}/dr^* > 0$ because $R^{SB} > \hat{\theta}(R^{SB})$ and $dr_M/dr^* < 0$ by (A.40).

□

Proof of Proposition 8. Solving (34) for θ yields

$$\theta \geq \frac{\tilde{L}(1+r) - R^{SB}}{1+r} - (1-\alpha)(\tilde{I}-\tilde{w})\frac{1+r_D}{1+r}(r-r_D) =: \hat{\theta}. \quad \text{(A.46)}$$

We differentiate (A.46) with respect to α and obtain

$$\frac{\partial\hat{\theta}}{\partial\alpha} = (\tilde{I}-\tilde{w})\frac{1+r_D}{1+r}(r-r_D) > 0, \quad \text{(A.47)}$$

which concludes the proof. □

Proof of Proposition 9. From equation (35) it follows directly that the quality threshold applied by the bank is given by

$$\hat{\theta}(\alpha) = \frac{\tilde{L}}{\tilde{\gamma}} - \frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha}. \tag{A.48}$$

The change in the threshold due to a change in α is

$$\frac{d\hat{\theta}}{d\alpha} = \frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha^2} > 0. \tag{A.49}$$

Finally, note that

$$\begin{aligned}\frac{dZ}{d\alpha} &= -f(\hat{\theta})\frac{d\hat{\theta}}{d\alpha} \\ &= -f(\hat{\theta})\,\frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha^2} < 0.\end{aligned} \tag{A.50}$$

□

Proof of Proposition 10. The equilibrium loan contract under perfect bank competition solves

$$\max_{d,R} \quad \pi_E(d,R)$$

subject to

$$\pi_B(d,R) \geq 0, \tag{PC}$$

$$d \leq w = 0, \tag{LL}$$

and taking into account that $\hat{\theta}(R) = (L-R)/\gamma$. In equilibrium, banks make a zero profit. Solving (PC) as an equality for d and inserting this into the target function yields

$$F(\hat{\theta}(R))L + (\gamma+1)\int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - c - I. \tag{A.51}$$

Thus, if (LL) is slack, the equilibrium contract maximizes the joint surplus of the entrepreneur and the bank. This is achieved for

$$R = \frac{L}{1+\gamma} = \theta^*.$$

The corresponding part financed by the entrepreneur is

$$\bar{d} = -F(\theta^*)L - \gamma\int_{\theta^*}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1-F(\theta^*)]\theta^* + c + I. \tag{A.52}$$

Thus, for $\bar{d} \leq 0$, the equilibrium loan contract is $(\bar{d}, R^*)$ and the first-best allocation is implemented.

For $\bar{d} > 0$, the contract $(\bar{d}, R^*)$ is not feasible. The highest feasible d is $d = 0$. Now, the repayment needs to be increased in order to satisfy (PC). Thus, the equilibrium repayment R^C solves $\pi_B(0, R^C) = 0$. Note that $\pi_B(0, R)$ is strictly increasing in $R \leq \bar{R}$. Moreover, by Assumption 4, $\pi_B(0, \bar{R}) > 0$. As a result, there exists a unique $R^C \in (R^*, \bar{R})$ that solves $\pi_B(0, R^C) = 0$. Finally, as $\pi_E(0, R)$ is strictly decreasing in $R \in [R^*, \bar{R}]$ and $\pi_E(0, \bar{R}) = 0$, we know that $\pi_E(0, R^C) > 0$, implying that the entrepreneur accepts a loan contract $(0, R^C)$.

□

References

Acharya, V. V., Borchert, L., Jager, M. and Steffen, S. (2021a). Kicking the can down the road: government interventions in the european banking sector. *The Review of Financial Studies*, **34** (9), 4090–4131.

—, Crosignani, M., Eisert, T. and Eufinger, C. (2020). *Zombie Credit and (Dis-)Inflation: Evidence from Europe*. Working Paper 27158, National Bureau of Economic Research.

—, —, — and Steffen, S. (2021b). Zombie lending: Theoretical, international, and historical perspectives. *CEPR Discussion Paper No. DP16685*.

—, Eisert, T., Eufinger, C. and Hirsch, C. (2019). Whatever it takes: The real effects of unconventional monetary policy. *The Review of Financial Studies*, **32** (9), 3366–3411.

—, Lenzu, S. and Wang, O. (2021c). *Zombie Lending and Policy Traps*. Working Paper 29606, National Bureau of Economic Research.

Adalet McGowan, M., Andrews, D. and Millot, V. (2017). The walking dead? zombie firms and productivity performance in oecd countries. *OECD Economics Department Working Papers*, **1372**.

Andrews, D. and Petroulakis, F. (2019). *Breaking the Shackles: Zombie Firms, Weak Banks and Depressed Restructuring in Europe*. Available at ssrn: `https://ssrn.com/abstract=3957694`, European Central Bank: ECB Working Paper.

Aragon, N. (2022). *Debt Overhang, Risk Shifting and Zombie Lending*. Working Paper 1/2022, National Bank of Ukraine.

Banerjee, R. and Hofmann, B. (2018). The rise of zombie firms: causes and consequences. *BIS Quarterly Review*.

— and — (2021). *Corporate zombies: Anatomy and life cycle*. Bis working papers, no 882, Bank for International Settlements,.

Beer, C., Ernst, N. and Waschiczek, W. (2021). The share of zombie firms among Austrian nonfinancial companies. *Monetary Policy & the Economy*, **Q2/21**, 35–58.

Bittner, C., Fecht, F. and Georg, C.-P. (2021). Contagious zombies. *Discussion Paper, Deutsche Bundesbank*.

Blattner, L., Farinha, L. and Rebelo, F. (forthcoming). When losses turn into loans: The cost of undercapitalized banks. *American Economic Review*.

Blažková, I. and Chmelíková, G. (2022). Zombie firms during and after crisis. *Journal of Risk and Financial Management*, **15** (7).

Borio, C. (2018). A blind spot in today's macroeconomics. *Bank for International Settlements: BIS management speeches*.

Bruche, M. and Llobet, G. (2014). Preventing zombie lending. *The Review of Financial Studies*, **27** (3), 923–956.

Caballero, R. J., Hoshi, T. and Kashyap, A. K. (2008). Zombie lending and depressed restructuring in japan. *American Economic Review*, **98** (5), 1943–77.

Chari, A., Jain, L. and Kulkarni, N. (2021). *The Unholy Trinity: Regulatory Forbearance, Stressed Banks and Zombie Firms*. No. w28435, National Bureau of Economic Research.

Daniel, K., Garlappi, L. and Xiao, K. (2021). Monetary policy and reaching for income. *The Journal of Finance*, **76** (3), 1145–1193.

De Martiis, A. and Peter, F. J. (2021). *When Companies Don't Die: Analyzing Zombie and Distressed Firms in a Low Interest Rate Environment*. Available at ssrn 3890788, University of Bern.

Diamond, D. W. (1991). Monitoring and reputation: The choice between bank loans and directly placed debt. *Journal of Political Economy*, **99** (4), 689–721.

Domian, D. L., Gilster, J. E. and Louton, D. A. (1996). Expected inflation, interest rates, and stock returns. *Financial Review*, **31** (4), 809–830.

Drechsler, I., Savov, A. and Schnabl, P. (2018). A model of monetary policy and risk premia. *Journal of Finance*, **73** (1), 317–373.

Faria-e-Castro, M., Paul, P. and Sanchez, J. M. (2021). *Evergreening*. Working Papers 2021-012, Federal Reserve Bank of St. Louis.

Giannetti, M. and Simonov, A. (2013). On the real effects of bank bailouts: Micro evidence from japan. *American Economic Journal: Macroeconomics*, **5** (1), 135–67.

Göbel, M. and Tavares, N. (2022). *Zombie-Lending in the United States–Prevalence versus Relevance*. Working paper – arxiv preprint arxiv:2201.10524, Universidade de Lisboa.

Gouveia, A. F. and Osterhold, C. (2018). Fear the walking dead: zombie firms, spillovers and exit barriers. *OECD Productivity Working Papers*, **13**.

Hoshi, T. (2000). Naze nihon wa ryudosei no wana kara nogarerarenainoka? (why is the japanese economy unable to get out of a liquidity trap?). In M. Fukuo and H. Yoshikawa (eds.), *Zero Kinri to Nihon Keizai (Zero Interest Rate and the Japanese Economy)*, Tokyo: Nihon Keizai Shimbunsha, pp. 233–266.

Hu, Y. and Varas, F. (2021). A theory of zombie lending. *Journal of Finance*, **76** (4), 1813–1867.

James, C. (1987). Some evidence on the uniqueness of bank loans. *Journal of Financial Economics*, **19** (2), 217–235.

Jaskowski, M. (2015). Should zombie lending always be prevented? *International Review of Economics & Finance*, **40**, 191–203.

Jordà, Ò., Kornejew, M., Schularick, M. and Taylor, A. M. (2022). Zombies at Large? Corporate Debt Overhang and the Macroeconomy. *The Review of Financial Studies*, **35** (10), 4561–4586.

Kwon, H. U., Narita, F. and Narita, M. (2015). Resource reallocation and zombie lending in japan in the 1990s. *Review of Economic Dynamics*, **18** (4), 709–732.

Laeven, L., Scheppens, G. and Schnabel, I. (2000). Zombification in europe in times of pandemic. VOXeu `https://voxeu.org/article/zombification-europe-times-pandemic`.

Leland, H. E. and Pyle, D. H. (1977). Informational asymmetries, financial structure, and financial intermediation. *Journal of Finance*, **32** (2), 371–387.

Ma, Z., Stice, D. and Williams, C. (2019). The effect of bank monitoring on public bond terms. *Journal of Financial Economics*, **133** (2), 379–396.

Peek, J. and Rosengren, E. S. (2005). Unnatural selection: Perverse incentives and the misallocation of credit in japan. *American Economic Review*, **95** (4), 1144–1166.

Puri, M. (1999). Commercial banks as underwriters: Implications for the going public process. *Journal of Financial Economics*, **54** (2), 133–163.

Rajan, R. G. (1994). Why bank credit policies fluctuate: A theory and some evidence. *Quarterly Journal of Economics*, **109** (2), 399–441.

Schivardi, F., Sette, E. and Tabellini, G. (2021). Credit misallocation during the european financial crisis credit misallocation during the crisis. *Economic Journal*, **132** (641), 391–423.

Storz, M., Koetter, M., Setzer, R. and Westphal, A. (2017). Do we want these two to tango? on zombie firms and stressed banks in europe. *ECB Working Paper*.

Tracey, B. (2021). The real effects of zombie lending in europe. *Bank of England, Staff Working Paper*.