ENVIRONMENTAL TAXATION AND FINANCIAL FRICTIONS IN GREEN LENDING

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ABSTRACT. We study polluting firms that require loans from a monopolistic bank to invest in abatement technology. Firms differ in the effectiveness of abatement investment, and this effectiveness is private information. The bank offers a screening contract under which high-cost firms receive too little capital and therefore emit excessively. A regulator restricted to tax policy responds by setting an environmental tax above marginal environmental damage, i.e., above the Pigouvian level. The first-best allocation can be restored by combining the Pigouvian tax, which ensures efficient abatement, with tailored, type-specific loan subsidies that correct the credit-market distortion.

Keywords: Abatement investment; Asymmetric information; Environmental taxation; Financial Frictions; Screening
JEL Codes: D82; G21; H23; Q58

1. INTRODUCTION

Meeting climate targets – most prominently under the Paris Agreement (United Nations Framework Convention on Climate Change, 2015) and, in Europe, the legally binding objective of climate neutrality by 2050 (European Union, 2021) – requires substantial upfront investment by private firms in cleaner production processes and abatement technologies (International Energy Agency, 2025; Evans et al., 2009). At the same time, financial flows toward climate mitigation and adaptation remain below estimated needs, implying a sizable climate-finance gap (Climate Policy Initiative, 2025; IPCC, 2022; Kapeller et al., 2023). Since firms may not be able to fully finance these investments internally, access to external funding becomes a key determinant of whether emissions-reducing investment is undertaken at scale. Indeed, a growing empirical literature shows that tighter financing conditions reduce firms' investment in cleaner technologies and increase emission intensity (see, for example, Accetturo et al., 2024; Andersen, 2017; De Haas and Popov, 2023; De Haas et al., 2025; Martinsson et al., 2024).[1] Moreover, many of these

Date: July 14, 2026.
Acknowledgments: We thank Henrik Guhling, conference audiences at Bavarian Micro Day (2026 Munich), and seminar audiences at the University of Bayreuth for helpful comments and suggestions.

[1] Related work links limited access to finance to greenwashing (Zhang, 2022), documents that existing financial policies may fail to alleviate frictions for green investment or may even hamper green innovation (Chai et al., 2022; Dafermos and Nikolaidi, 2021; Kedward et al., 2022; Lin and Pan, 2023; Liu et al.,

firms—particularly small and medium-sized enterprises—have limited access to market-based finance and therefore depend on bank loans, often provided by their house bank (Ferrando et al., 2023; Nerlich et al., 2025). These findings suggest that environmental policy cannot be designed independently of firms' access to external finance.

This raises a natural question: how should environmental policy, and in particular emissions taxation and subsidies for green loans, be designed when firms' abatement investment decisions are shaped by imperfect financial markets?

We address this question in a contract-theoretic model. Polluting firms can reduce emissions by investing in abatement technology, which must be financed through a loan from their house bank. The house-bank relationship gives the lender market power, allowing us to model the bank as a monopolistic supplier of credit. Firms differ in the effectiveness of abatement investment: for low-cost firms, investment substantially reduces abatement costs, whereas for high-cost firms it is less effective. A firm's cost type is private information. Under symmetric information, the Pigouvian tax implements the efficient allocation, inducing optimal abatement and efficient capital provision for both types. Under asymmetric information, the bank offers a menu of loan contracts to screen firms. Low-cost firms receive the efficient level of capital, given the tax-induced abatement incentives, while high-cost firms receive too little capital and therefore emit excessively; they face constrained credit provision. A regulator restricted to tax policy responds by setting the environmental tax above marginal environmental damage, i.e., above the Pigouvian level. Although this distorts abatement for given capital levels, it relaxes the underprovision of credit to high-cost firms by increasing their borrowing. However, it also expands lending to low-cost firms and distorts their abatement incentives, so a tax-only policy cannot implement the first best. Combining the tax with a uniform loan subsidy improves welfare but remains insufficient. The first-best allocation can be achieved only by pairing the Pigouvian tax with type-specific loan subsidies: the tax ensures efficient abatement incentives, while the targeted subsidies correct the type-specific distortions in capital provision.[2] The focus on environmental taxation and loan subsidies resonates with survey

2021), and shows that environmental regulation can have unintended side effects when firms are financially constrained (Bartram et al., 2022). Further empirical evidence includes Andersen (2016, 2020); Ang et al. (2017); Bellon and Boualam (2024); Best (2017); Brunnschweiler (2010); Costa et al. (2024); Kalantzis et al. (2022); Kim and Park (2016); Levine et al. (2021); Ng et al. (2023); Qin et al. (2024); Sfrappini (2024); Zhang and Vigne (2021). Some evidence points in the opposite direction, showing that financially constrained firms may reduce emissions more than unconstrained firms (Guerini et al., 2025).

[2]In our model, environmental taxation and loan subsidies are complements. By contrast, environmental taxation and pollution-dependent interest rates are substitutes in Pedersen (2026).

evidence from Stroebel and Wurgler (2021), who document that finance academics, professionals, and regulators regard these instruments as among the most important tools for reducing firms' carbon footprints.

Our analysis first speaks to work on firm heterogeneity and investment in green or advanced abatement technologies. Existing contributions typically emphasize differences in output productivity. In Forslid et al. (2018) and Cui et al. (2016), more productive firms are more likely to invest in cleaner technologies and to export, which lowers their emissions intensity. Cao et al. (2016) show that the relationship between productivity and green investment depends on whether productivity and investment are complements or substitutes. We depart from this literature by shifting the relevant dimension of heterogeneity from output productivity to the effectiveness of abatement investment itself. Firms in our model differ in how strongly capital investment lowers their abatement costs. This allows us to study how differences in abatement effectiveness interact with credit-market screening and, ultimately, with environmental policy.

Second, our paper relates to the literature studying how environmental-policy instruments affect firms' incentives to invest in green technologies.[3] Requate and Unold (2001) show that, when the regulator must commit to a policy before knowing which technologies will become available in the future, environmental taxes may induce overinvestment, whereas tradable permits may induce underinvestment. If the regulator knows the advanced technology, both instruments can implement the first-best outcome. Similar findings are obtained by Denicolò (1999), who studies a monopolistic innovator licensing an environmental innovation to polluting firms. D'Amato and Dijkstra (2015) instead analyze a strategic polluting firm and show that, under policy commitment, tradable permits may create stronger incentives to invest in green technology than environmental taxes. More recently, Biais and Landier (2022) study a dynamic model with investment spillovers and a regulator who lacks commitment power. Their model features multiple equilibria. In a good equilibrium, firms invest early, thereby lowering aggregate abatement costs in the future and inducing stringent future regulation. In a bad equilibrium, firms do not invest because they expect the regulator to set a lenient emissions cap in the future. They further show that financial frictions expand the parameter region in which the bad equilibrium arises. The central mechanisms in these papers are the timing of environmental policy, uncertainty about future technologies, or environmental damages. By contrast, we consider a regulator with full commitment power and knowledge of aggregate

[3]We also contribute to the literature on directed technological change (Aghion et al., 2016; Acemoglu et al., 2016). A model of directed technological change with financial frictions is analyzed by Campiglio et al. (2024), who show that the efficient allocation can be implemented by combining environmental taxation with a set of subsidies.

abatement costs and marginal environmental damages. The relevant informational friction in our model instead arises in the credit market: firms privately know how effectively they can use abatement investment, and the bank must screen them through loan contracts. We share with this literature the benchmark insight that, absent additional frictions beyond the environmental externality, a Pigouvian tax equal to marginal environmental damage can induce both efficient abatement and efficient investment.[4]

Third, our paper contributes to the recent and growing literature on the interaction between environmental policy and financial frictions.[5] A first set of papers models financial frictions as moral-hazard or financing-capacity constraints between lenders and polluting firms. Hoffmann et al. (2017) study a setting in which moral hazard limits the final repayment that is compatible with firms exerting high effort and, hence, with a high probability of successful output. A more stringent environmental tax exacerbates this moral-hazard problem, implying that the optimal environmental tax lies below marginal environmental damage. Inderst and Heider (2023) build on the workhorse model of Holmström and Tirole (1997), in which firms' financing capacities are limited by available internal funds. They analyze an industry with green and brown firms and show that, if the industry is not financially constrained in the aggregate, the optimal environmental tax may exceed marginal environmental damage. Döttling and Rola-Janicka (2025) add physical climate risk affecting asset values. Similar to Hoffmann et al. (2017), they find that, absent physical risk, the optimal environmental tax is below marginal environmental damage. With physical risk, however, the optimal tax may exceed the Pigouvian level. Additionally, Döttling and Rola-Janicka (2025) and Inderst and Heider (2023) show that, under emissions trading, the allocation mechanism is not welfare neutral: grandfathering can relax financial constraints relative to permit auctions. More closely related to our paper are contributions that model the financial friction as a screening problem between lenders and borrowers. Haas and Kempa (2023) analyze a market with skilled and unskilled firms that require external finance to invest in a low-emission technology. Due to asymmetric information, some firms that should receive capital from a welfare perspective do not obtain financing and therefore do not adopt the green technology. In their framework, an environmental tax above the Pigouvian level may be sufficient to implement the first-best allocation. If it is not, Pigouvian taxation combined with a loan subsidy restores efficiency. In Haas and Kempa (2023), however, emissions depend on technology choice but

[4]Ahlvik and van den Bijgaart (2024) study a setting in which firms differ in innovation quality, which is private information and therefore unobserved by the regulator. They show that, when innovation subsidies cannot be tailored to innovation quality, the second-best policy combines an environmental tax above marginal environmental damages with the granting of patent rights.

[5]An orthogonal question regarding the taxation of externalities under financial frictions is analyzed by Tirole (2010), who studies a model in which the polluter may be unable to pay for the harm caused.

not on a subsequent abatement decision. Herweg (2025) studies a related model in which firms decide whether to adopt a green technology and, after adoption, choose abatement. Firms adopting the green technology require a loan from a bank that charges an intermediation margin. The optimal tax-only policy exceeds the Pigouvian level, whereas a Pigouvian tax combined with a loan subsidy implements the first-best outcome. We depart from these papers by allowing firms to make both a continuous investment decision in advanced abatement technology and a continuous abatement decision. This distinction matters for policy design. In our model, only an environmental tax equal to marginal environmental damage implements efficient abatement choices for given investment levels. At the same time, asymmetric information in the credit market creates type-dependent distortions in capital provision. As a result, a single additional uniform credit instrument is generally insufficient to restore efficiency. Instead, the first-best allocation requires separate instruments for separate distortions: the Pigouvian tax implements efficient abatement incentives, while type-specific loan subsidies correct the type-dependent distortions in investment. In this sense, the Tinbergen (1952) rule applies: each independent policy target requires its own policy instrument.

The remainder of the paper is organized as follows. Section 2 introduces the model and characterizes the first-best allocation. Section 3 first examines the full-information benchmark and then analyzes the optimal bank contract under asymmetric information. Section 4 studies alternative policy regimes: a tax-only policy, an environmental tax combined with a uniform loan subsidy, and an environmental tax combined with tailored loan subsidies. Section 5 extends the analysis to a continuum of cost types and shows that the main findings are robust. Section 6 concludes.

2. The Model

2.1. **Polluting Firms.** We analyze a static, one-period economy in which a continuum of polluting firms (measure one) seeks external finance from a monopolistic bank. Each firm is characterized by a type $\theta \in \{\theta_L, \theta_H\}$, with $0 < \theta_L < \theta_H$, that is a firm's private information. A fraction $\nu \in (0,1)$ of firms are of the type θ_L, while the remaining share $1-\nu$ are of the θ_H-type. In the business-as-usual scenario, each firm emits $q^B > 0$ units of pollution and earns a revenue $\Pi > 0$ at the end of the period. Each unit of pollution generates an environmental damage of $\kappa > 0$, implying constant marginal social costs of pollution. The government regulates emissions through a per-unit environmental tax $\tau \geq 0$. Accordingly, under the business-as-usual scenario, a firm's profit equals

$$\bar{V}_\tau = \Pi - \tau q^B, \tag{1}$$

which serves as the firm's outside option for any given tax level.

A firm can reduce its emissions – and thereby its environmental tax burden – by investing in an abatement technology. If a firm of type θ invests an amount $k > 0$ in abatement technology, its abatement cost is

$$\frac{\theta}{k}C(\Delta), \tag{2}$$

where $\Delta = q^B - q$ denotes the amount of abatement, and $q \in [0, q^B]$ represents the firm's actual emissions. We refer to type θ_H as the high-cost type and to type θ_L as the low-cost type. Regarding the 'abatement cost function' $C(\Delta)$, we impose the following two assumptions.

Assumption 1. *The 'abatement cost function' $C(\cdot)$ satisfies:*

(i) $C(0) = 0$ *and* $C'(\Delta) > 0$ *for* $\Delta > 0$*;*
(ii) $C''(\Delta) > 0$ *for all* $\Delta > 0$*;*
(iii) $C'(0) = 0$ *and* $\lim_{\Delta \to q^B} C'(\Delta) = +\infty$.

According to Assumption 1(i), abatement is costly, while the absence of abatement entails no cost. Assumption 1(ii) states that the marginal cost of abatement increases with the level of abatement; in other words, reducing emissions becomes progressively more expensive as emissions become lower. Finally, Assumption 1(iii) imposes Inada-type conditions that ensure interior solutions for abatement and emission levels.

In addition, we impose the following technical assumption to simplify the subsequent analysis.

Assumption 2. *The 'abatement cost function' $C(\cdot)$ satisfies: For all $\Delta > 0$,*

$$\frac{2C''(\Delta)}{C'(\Delta)} > \frac{C'(\Delta)}{C(\Delta)} > \frac{C''(\Delta)}{C'(\Delta)}. \tag{3}$$

The first inequality in Assumption 2 guarantees that the welfare function is globally concave. The second inequality ensures that the low-cost type, θ_L, benefits more from an increase in capital than the high-cost type, θ_H. This property corresponds to the standard Spence-Mirrlees single-crossing condition (Spence, 1974; Mirrlees, 1971). As a leading example that satisfies Assumptions 1 and 2, we employ the cubic abatement cost function $C(\Delta) = \Delta^3/3$.[6]

[6]The second part of Assumption 1(iii), $\lim_{\Delta \to q^B} C'(\Delta) = +\infty$, is not satisfied by the cubic specification. In our illustrative example, we therefore impose parameter restrictions that ensure interior solutions. More generally, any power abatement cost function $C(\Delta) = \Delta^\alpha/\alpha$ with $\alpha > 2$ could serve as the leading example. This restriction rules out the quadratic case: for a quadratic abatement cost function, the first part of Assumption 2 fails, so the welfare function does not have an interior maximizer.

2.2. **Bank Contract and Timing.** Each firm is initially cashless and therefore requires external financing – specifically, a bank loan – to invest in abatement technology. We adopt a standard loan-contracting setup with screening under adverse selection (Freixas and Laffont, 1990; Laffont and Martimort, 2009): the monopolistic bank offers a menu of loan contracts $\langle (k_i, R_i)_{i=L,H} \rangle$ where each contract specifies the amount of capital provided, k_i, and the corresponding repayment, R_i.[7] While a firm's cost type is private information, the bank knows that firms differ in their cost types and is aware of their distribution in the economy. Consequently, the bank may design a screening menu that includes a distinct loan contract for each type θ_i, with $i = L, H$. The bank faces constant per-unit capital costs of $\rho > 0$, where $\rho = 1 + r$ and r denotes the risk-free refinancing rate.

The timing of the game is as follows:

1. The government specifies its environmental policy; in particular, it sets the environmental tax $\tau \geq 0$.
2. The bank offers a menu of loan contracts to the firms. Each firm decides whether or not to accept one of the loan contracts.
3. Firms that have signed a loan contract invest the borrowed amount in abatement technology.
4. Each firm chooses its emission level q and produces. Firms that did not invest in abatement choose the business-as-usual emission level q^B.
5. Each firm generates revenue Π, pays its environmental taxes, and, in the case of a bank loan, makes the repayment to the bank.

We assume that the revenue $\Pi > 0$ is sufficiently large for each firm to meet its obligations (tax and loan) at the end of the game. In other words, we abstract from any shut-down decisions.

2.3. **Welfare and First-Best Allocation.** The social surplus (or welfare) generated by a firm of type θ that receives capital $k > 0$ and emits q units of pollution is given by

$$W_\theta(k, q) = \Pi - \frac{\theta}{k} C(q^B - q) - \rho k - \kappa q. \tag{4}$$

The firm earns revenue Π and incurs abatement costs $(\theta/k)C(\Delta)$. Providing capital entails the resource cost ρk, corresponding to the bank's cost of funds and hence the social cost of capital. Emissions generate environmental damages κq, representing the social cost of the pollution caused.

[7] We assume that the bank cannot write sophisticated contracts conditioning on a firm's emissions (or, equivalently, that the bank cannot observe emissions).

An omniscient social planner chooses $(k^*(\theta), q^*(\theta))$ for each firm type $\theta \in \{\theta_L, \theta_H\}$. We assume that the welfare-maximizing capital levels for both types are strictly positive. By Assumption 2, the welfare function (4) is globally concave, implying that the first-best allocation satisfies the following first-order conditions:

$$\frac{\theta}{k^*(\theta)^2} C(q^B - q^*(\theta)) = \rho, \tag{5}$$

$$\frac{\theta}{k^*(\theta)} C'(q^B - q^*(\theta)) = \kappa. \tag{6}$$

For conciseness, let $k_i^* := k^*(\theta_i)$ and $q_i^* := q^*(\theta_i)$ for $i \in \{L, H\}$. It follows directly from (5)–(6) that the low-cost type receives more capital and abates more (i.e., pollutes less) than the high-cost type,

$$k_L^* > k_H^*, \quad q_L^* < q_H^*.$$

Furthermore, the first-best capital allocation decreases with the cost of capital and increases with the marginal environmental damage. The comparative statics for optimal pollution levels are reversed: efficient pollution increases with the cost of capital and decreases with the marginal environmental damage:

$$\frac{dk^*}{d\rho} < 0, \quad \frac{dk^*}{d\kappa} > 0, \quad \frac{dq^*}{d\rho} > 0, \quad \frac{dq^*}{d\kappa} < 0. \tag{7}$$

Remarks on Modelling Assumptions. A few remarks on the modelling assumptions are in order. We model the financial friction purely as the distortion caused by monopolistic screening and thereby rule out default risk for the bank. While our modelling approach follows Freixas and Laffont (1990), this is somewhat unusual in the microeconomics of banking literature (Freixas and Rochet, 2008). Adding default risk to our model would not affect the key findings as long as such risk does not interact with the effectiveness of capital provision in reducing abatement costs. Nevertheless, because of this assumption, our model is in some respects similar to one in which a monopolistic provider of a scalable abatement technology sells this technology to firms that differ in its effectiveness.

A second point worth mentioning is that we restrict bank contracts to specify a credit volume and a final repayment, or equivalently an interest rate on the provided credit. While such contracts resemble real-world loan contracts, an optimal mechanism in our setup may condition on the pollution level, which is observed at least by the regulator, thereby potentially reducing or even eliminating the screening problem. In other words, we implicitly assume that making final repayments depend on pollution intensity is not feasible.

3. The Analysis

3.1. **Full Information.** We begin the analysis with the benchmark case of full information, in which the bank observes each firm's cost type and offers a credit contract conditional on the observed type. For a given cost type $\theta \in \{\theta_L, \theta_H\}$, the bank offers a loan contract $(k(\theta), R(\theta))$ to maximize its profit,

$$B_\theta(k, R) = R(\theta) - \rho\, k(\theta). \tag{8}$$

At the end of the period, the bank receives the repayment $R(\theta)$, having incurred capital costs of $\rho k(\theta)$. When designing the loan contract, the bank must ensure that the contract is sufficiently attractive for the firm to accept it; that is, the participation constraint must be satisfied.

To specify the participation constraint for a given cost type, we first analyze a firm's abatement decision upon accepting a loan contract (k, R). If a firm of type θ accepts the contract (k, R), it invests the amount k in the abatement technology and chooses the emission level that minimizes the sum of abatement and tax costs:

$$\frac{\theta}{k} C(q^B - q) + \tau q. \tag{9}$$

The cost-minimizing emission level $\hat{q}(z)$, with $z := \tau k/\theta$, and thus the corresponding abatement level $\hat{\Delta} = q^B - \hat{q}$, is implicitly characterized by

$$C'(q^B - \hat{q}(z)) = z. \tag{10}$$

For $z > 0$, differentiating with respect to z yields

$$\frac{d\hat{q}}{dz} = -\frac{1}{C''(q^B - \hat{q}(z))} < 0. \tag{11}$$

Hence, the cost-minimizing emission level decreases – and therefore abatement increases – with the environmental tax τ and the amount of capital invested k, while emissions increase (and abatement decreases) with the firm's cost type θ. The profit earned by a firm of type θ that accepts a loan contract (k, R) is given by

$$\tilde{V}_\tau(k, R \mid \theta) = \Pi - \frac{\theta}{k} C(q^B - \hat{q}) - \tau \hat{q} - R. \tag{12}$$

A firm of type θ accepts the offered loan contract $(k(\theta), R(\theta))$ if the resulting profit $\tilde{V}_\tau$ weakly exceeds its outside option $\bar{V}_\tau$. Equivalently, the net advantage of type θ from accepting the contract, $\tilde{V}_\tau - \bar{V}_\tau$, is given by

$$V(\theta) = \tau q^B + \left[-\tau \hat{q}\left(\frac{\tau k(\theta)}{\theta}\right) - \frac{\theta}{k(\theta)} C\left(q^B - \hat{q}\left(\frac{\tau k(\theta)}{\theta}\right)\right)\right] - R(\theta), \tag{13}$$

and the participation constraint requires that

$$V(\theta_i) \geq 0 \tag{PC$_i$}$$

for $i \in \{L, H\}$.

In the optimum, the bank specifies a loan contract such that the participation constraint (PC_i) is satisfied with equality. Hence, for each type θ, the bank chooses the amount of capital $k(\theta)$ to be lent in order to maximize the following profit expression:

$$B_\theta(k(\theta)) = \tau \left[q^B - \hat{q}\left(\frac{\tau k(\theta)}{\theta}\right)\right] - \frac{\theta}{k(\theta)} C\left(q^B - \hat{q}\left(\frac{\tau k(\theta)}{\theta}\right)\right) - \rho k(\theta). \tag{14}$$

For ease of exposition we assume that for any $\tau > 0$ the bank finds it optimal to lend to both types under full information.

Lemma 1. *Under full information, the bank offers to each type θ_i, $i \in \{L, H\}$, a loan contract $(k_\tau^{FB}(\theta_i), R_\tau^{FB}(\theta_i))$, where $k_\tau^{FB}(\theta_i)$ solves*

$$\frac{\theta_i}{\left(k_\tau^{FB}(\theta_i)\right)^2} C\left(q^B - \hat{q}\left(\frac{\tau\, k_\tau^{FB}(\theta_i)}{\theta_i}\right)\right) = \rho, \tag{15}$$

and $R_\tau^{FB}(\theta_i)$ is chosen such that the participation constraint holds with equality, i.e., firms receive no (informational) rents.

Proof. All proofs are deferred to Appendix A. □

We denote the full-information solution by the superscript FB (first-best), even though it does not represent the welfare-optimal allocation. It is, however, the optimal allocation from a purely contractual perspective – that is, it maximizes the joint surplus of the bank and the firm.

A comparison of the first-order conditions for individual optimization by firms and banks, (10) and (15), with those of the omniscient social planner, (5) and (6), reveals that under full information the provided capital and abatement decisions are efficient if the environmental tax equals the marginal environmental damage. In other words, the Pigouvian tax $\tau^* = \kappa$ implements the efficient allocation.

Proposition 1. *Under full information, the Pigouvian tax $\tau^* = \kappa$ implements the welfare-optimal outcome. Specifically, it ensures for every type θ_i, $i \in \{L, H\}$, both (i) optimal emissions (equivalently, optimal abatement) and (ii) optimal capital provision:*

$$q_i^* = \hat{q}\left(\frac{\kappa\, k_\kappa^{FB}(\theta_i)}{\theta_i}\right), \qquad k_i^* = k_\kappa^{FB}(\theta_i).$$

Under full information, where the bank can extract all rents from the served firms, a single policy instrument – an environmental tax – is sufficient to implement the welfare-optimal allocation. If the environmental tax equals the marginal environmental damage, each firm chooses the efficient abatement level (given the provided capital). Since the joint surplus is then maximized at the efficient capital level and firms receive no rents, the bank has an incentive to provide the efficient amount of capital.

3.2. **Asymmetric Information.** We now turn to the case of asymmetric information, where the cost type θ is private information of the firm. The bank offers a screening menu $\langle (k_L, R_L), (k_H, R_H) \rangle$, where contract (k_i, R_i) is intended for type θ_i. As the bank cannot observe firms' cost types, the different types must self-select into the correct contracts; that is, the bank must take the incentive compatibility constraints into account when designing its contract menu.

Incentive compatibility requires that neither the high-cost type θ_H nor the low-cost type θ_L has an incentive to choose the contract intended for the other type. Formally,

$$V(\theta_H) \geq V(\theta_L) - \psi(k_L), \tag{IC$_H$}$$

$$V(\theta_L) \geq V(\theta_H) + \psi(k_H), \tag{IC$_L$}$$

where

$$\begin{aligned} \psi(k) \equiv \tau\, \hat{q}\left(\frac{\tau k}{\theta_H}\right) + \frac{\theta_H}{k}\, C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta_H}\right)\right) \\ - \tau\, \hat{q}\left(\frac{\tau k}{\theta_L}\right) - \frac{\theta_L}{k}\, C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta_L}\right)\right) > 0 \end{aligned} \tag{16}$$

is the difference between the total costs of the high-cost and low-cost types. Hence, neither type has an incentive to mimic the other if and only if

$$\psi(k_L) \geq V(\theta_L) - V(\theta_H) \geq \psi(k_H). \tag{17}$$

The chain of inequalities (17) implies that $V(\theta_L) \geq V(\theta_H)$ and $k_L \geq k_H$, since $\psi'(k) > 0$ by Assumption 2. In other words, under an incentive-compatible menu of loan contracts, the low-cost type obtains (weakly) higher informational rents than the high-cost type, and the monotonicity constraint $k_L \geq k_H$ is satisfied.

The bank maximizes its profit, which is given by

$$B(k_L, k_H, R_L, R_H) = \nu\big[R_L - \rho\, k_L\big] + (1-\nu)\big[R_H - \rho\, k_H\big], \tag{18}$$

subject to (PC$_L$), (PC$_H$), (IC$_L$), and (IC$_H$). Recall that a mass ν of firms is of the low-cost type, while a mass $(1-\nu)$ of firms is of the high-cost type. The bank finds it optimal to leave no rent to the high-cost type, implying that (PC$_H$) binds and thus $V(\theta_H) = 0$. Moreover, under the optimal contract, the incentive compatibility constraint of the low-cost type, (IC$_L$), binds, implying that $V(\theta_L) = \psi(k_H)$. These observations allow the bank's optimization problem to be expressed as an unconstrained maximization problem in terms of the capital schedule alone.[8] The bank therefore solves

$$\begin{aligned}(19)\quad \max_{k_L \geq k_H} \nu &\left[\tau q^B - \tau \hat{q}\left(\frac{\tau k_L}{\theta_L}\right) - \frac{\theta_L}{k_L} C\left(q^B - \hat{q}\left(\frac{\tau k_L}{\theta_L}\right)\right) - \rho k_L - \psi(k_H)\right] \\ &+ (1-\nu)\left[\tau q^B - \tau \hat{q}\left(\frac{\tau k_H}{\theta_H}\right) - \frac{\theta_H}{k_H} C\left(q^B - \hat{q}\left(\frac{\tau k_H}{\theta_H}\right)\right) - \rho k_H\right].\end{aligned}$$

The following lemma summarizes the second-best optimal capital schedule.

Lemma 2. *Under asymmetric information, if ν is not too large, the bank serves both cost types and the second-best optimal capital schedule $k_i = k_\tau^{SB}(\theta_i)$ for $i \in \{L, H\}$ is characterized by:*

$$(20)\qquad \frac{\theta_L}{\left[k_\tau^{SB}(\theta_L)\right]^2} C\left(q^B - \hat{q}\left(\frac{\tau\, k_\tau^{SB}(\theta_L)}{\theta_L}\right)\right) = \rho,$$

$$(21)\qquad -\frac{\nu}{1-\nu}\,\psi'\big(k_\tau^{SB}(\theta_H)\big) + \frac{\theta_H}{\left[k_\tau^{SB}(\theta_H)\right]^2} C\left(q^B - \hat{q}\left(\frac{\tau\, k_\tau^{SB}(\theta_H)}{\theta_H}\right)\right) = \rho.$$

The low-cost type receives the full-information capital amount, whereas the high-cost type's capital allocation is distorted downwards relative to the full-information benchmark (i.e., a screening-induced capital distortion arises under the second-best contract):

$$k_\tau^{FB}(\theta_L) = k_\tau^{SB}(\theta_L), \qquad k_\tau^{FB}(\theta_H) > k_\tau^{SB}(\theta_H).$$

Under the second-best optimal loan contract, all firms of the low-cost type receive the amount of capital that maximizes the joint surplus of the bank and a low-cost firm (no distortion at the top). Moreover, if firms of the high-cost type are served, all low-cost firms obtain strictly positive information rents. The amount of capital provided to high-cost firms is distorted downwards relative to the full-information benchmark. In other words, the usual downward distortion at the bottom is observed.

From the above observations, the next result follows immediately.

Corollary 1. *Under asymmetric information, the Pigouvian tax $\tau^* = \kappa$ induces the following allocation:*

$$\begin{aligned} k_\kappa^{SB}(\theta_L) &= k_L^*, \qquad & \hat{q}\left(\frac{\kappa\, k_\kappa^{SB}(\theta_L)}{\theta_L}\right) &= q_L^*, \\ k_\kappa^{SB}(\theta_H) &< k_H^*, \qquad & \hat{q}\left(\frac{\kappa\, k_\kappa^{SB}(\theta_H)}{\theta_H}\right) &> q_H^*. \end{aligned}$$

[8]Strictly speaking, the optimization problem is not fully unconstrained, since the monotonicity constraint $k_H \leq k_L$ must be satisfied for incentive compatibility. For binary types, however, this monotonicity constraint never binds and thus does not restrict the solution.

Under Pigouvian taxation and asymmetric information, low-cost firms receive the efficient amount of capital and choose the efficient abatement level – that is, the efficient pollution level. High-cost firms, by contrast, receive too little capital from a welfare perspective and therefore abate too little, implying excessive pollution. Note, however, that under Pigouvian taxation, the emission level of high-cost firms is *constrained efficient* – that is, efficient given the too low amount of capital they receive from the bank.

3.3. **Example: Cubic Abatement Cost Function.** In the following, we consider an illustrative example that allows for closed-form solutions of all endogenous variables of interest. As explained above, our leading example is a cubic abatement cost function,

$$C(\Delta) = \frac{1}{3}\Delta^3. \tag{22}$$

Moreover, we assume that business-as-usual emissions $q^B > 0$ are sufficiently high so that welfare-optimal and equilibrium emission levels are interior.

It can readily be shown that the welfare-optimal emissions and capital levels are given by

$$q^*(\theta) = q^B - \frac{\kappa^2}{3\rho\theta}, \qquad k^*(\theta) = \frac{\kappa^3}{9\rho^2\theta}. \tag{23}$$

Hence, optimal abatement, $\Delta^*(\theta) = \kappa^2/(3\rho\theta)$, is independent of business-as-usual emissions q^B. The comparative statics are immediate: the high-cost type receives less capital and pollutes more than the low-cost type. Optimal emissions are decreasing in the marginal environmental damage κ and increasing in the cost of capital ρ. Conversely, the optimal capital level decreases in ρ and increases in κ.

Under individual optimization, for any environmental tax $\tau \geq 0$ and any amount of capital $k > 0$ received, a firm of type θ chooses the pollution level

$$\hat{q}\left(\frac{\tau k}{\theta}\right) = q^B - \sqrt{\frac{\tau k}{\theta}}, \tag{24}$$

and thus abates $\hat{\Delta}(\theta) = \sqrt{\tau k/\theta}$. In the full-information benchmark, where the bank observes each firm's type, optimal capital provision is given by

$$k_\tau^{FB}(\theta) = \frac{1}{9}\,\frac{\tau^3}{\rho^2\theta}. \tag{25}$$

Under asymmetric information, by contrast, the bank offers a menu of credit contracts. The capital levels of the second-best contract are given by

$$k_\tau^{SB}(\theta_L) = \frac{1}{9}\,\frac{\tau^3}{\rho^2\theta_L} \quad \text{and} \quad k_\tau^{SB}(\theta_H) = \frac{1}{9}\,\frac{\tau^3}{\rho^2\theta_H}\,\beta^2, \tag{26}$$

where

$$\beta := \frac{1}{1-\nu}\left(1-\nu\sqrt{\frac{\theta_H}{\theta_L}}\right) < 1, \tag{27}$$

provided that $\beta \geq 0$. The downward distortion in the amount of capital received by the high-cost type, relative to the full-information benchmark, is inversely related to β^2 and vanishes when $\beta = 1$. The distortion increases in the share of low-cost types ν and in the ratio between types, θ_H/θ_L. The condition $\beta \geq 0$, under which the solution for $k_\tau^{SB}(\theta_H)$ in (26) is valid, holds if and only if

$$\nu \leq \bar{\nu} := \sqrt{\frac{\theta_L}{\theta_H}}. \tag{28}$$

For $\nu > \bar{\nu}$, the bank optimally offers the null contract to the high-cost type, $(k_H, R_H) = (0,0)$, and thus effectively does not contract with that type. Importantly, it can be shown that the bank's profit function is globally concave in (k_L, k_H) if and only if $\nu \leq \bar{\nu}$ (it is always concave in k_L).

4. Optimal Policy

4.1. **Tax-Only Policy.** We begin the analysis of optimal (environmental) regulation by considering a regulator who is restricted to using an environmental tax as the sole policy instrument. We already know that a firm's abatement decision is efficient only if the environmental tax reflects the true marginal environmental damage, that is, under Pigouvian taxation $\tau^* = \kappa$. Under symmetric information between firms and the bank, the Pigouvian tax also induces the efficient provision of capital. Under asymmetric information, however, the Pigouvian tax no longer induces efficient capital provision for both types, as screening distorts the high-cost type's capital allocation downward. Thus, an optimal tax-only policy can at best implement a second-best outcome, trading off distortions in abatement against distortions in the capital levels provided to firms.

To analyze the optimal tax-only policy, we first examine how an increase in the environmental tax affects the capital levels allocated to firms under the second-best screening menu. As is apparent from the first-order condition (20), the amount of capital received by low-cost types is increasing in the environmental tax by Assumption 2, that is, $dk_\tau^{SB}(\theta_L)/d\tau > 0$. While it is intuitive that, with a higher tax and thus stronger benefits from investing in abatement technology, the amount of capital provided to low-cost firms increases, the effect on high-cost firms is less obvious. The reason is that it is, *a priori*, unclear whether a higher tax increases or decreases the incentive of low-cost firms to mimic high-cost firms; that is, whether an increase in the tax τ tightens or relaxes the binding incentive-compatibility constraint (IC$_L$). The sign of the derivative of the amount of capital received by high-cost types with respect to the tax can be determined unambiguously if the share of low-cost firms is not too high. In this case, it holds that $dk_\tau^{SB}(\theta_H)/d\tau > 0$. These observations are summarized in the following lemma.

Lemma 3. *Suppose that ν is not too high. Then the second-best contract assigns to each type θ_i, $i \in \{L, H\}$, an amount of capital that strictly increases with the environmental tax:*

$$\frac{dk_\tau^{SB}(\theta_L)}{d\tau} > 0 \quad \textit{and} \quad \frac{dk_\tau^{SB}(\theta_H)}{d\tau} > 0. \tag{29}$$

Recall that ν – the share of low-cost types – must be sufficiently low for high-cost types to receive a positive amount of capital under the second-best contract.[9]

The regulator chooses the environmental tax τ in order to maximize social surplus. The social surplus – i.e., welfare – as a function of τ, taking into account the optimal reactions of the bank and the firms, is given by

$$\begin{aligned}(30)\quad W(\tau) = &\\ &\nu\left[\Pi - \frac{\theta_L}{k_\tau^{SB}(\theta_L)} C\left(q^B - \hat{q}\left(\frac{\tau k_\tau^{SB}(\theta_L)}{\theta_L}\right)\right) - \kappa\hat{q}\left(\frac{\tau k_\tau^{SB}(\theta_L)}{\theta_L}\right) - \rho k_\tau^{SB}(\theta_L)\right]\\ +(1-\nu)&\left[\Pi - \frac{\theta_H}{k_\tau^{SB}(\theta_H)} C\left(q^B - \hat{q}\left(\frac{\tau k_\tau^{SB}(\theta_H)}{\theta_H}\right)\right) - \kappa\hat{q}\left(\frac{\tau k_\tau^{SB}(\theta_H)}{\theta_H}\right) - \rho k_\tau^{SB}(\theta_H)\right].\end{aligned}$$

A share ν of firms is of the low-cost type and a share $1-\nu$ of the high-cost type. Each firm generates revenue Π, incurs abatement costs, causes environmental damage, and induces costs of capital provision for the bank. Importantly, tax payments and tax revenues are welfare-neutral transfers. The environmental tax affects welfare only indirectly, through the induced reactions of firms and the bank.

We focus on the interesting case in which capital provision to high-cost firms is strictly positive, so that these firms adjust their behavior in response to changes in the environmental tax. Differentiating the welfare expression (30) with respect to τ, and substituting the first-order conditions of the bank, (20) and (21), as well as the firms' first-order condition (10), yields

$$\begin{aligned}(31)\quad W'(\tau) = (\tau - \kappa)\left\{\nu\left[\frac{\partial\hat{q}}{\partial\tau} + \frac{\partial\hat{q}}{\partial k}\frac{dk_\tau^{SB}(\theta_L)}{d\tau}\right] + (1-\nu)\left[\frac{\partial\hat{q}}{\partial\tau} + \frac{\partial\hat{q}}{\partial k}\frac{dk_\tau^{SB}(\theta_H)}{d\tau}\right]\right\}\\ + \nu\psi'(k_\tau^{SB}(\theta_H))\frac{dk_\tau^{SB}(\theta_H)}{d\tau}.\end{aligned}$$

If there were no screening-induced distortion in capital provision – that is, if $\psi'(\cdot) = 0$ – the first-order condition for welfare maximization, $W'(\tau) = 0$, would be satisfied at

[9] In the general model, we cannot rule out that the critical values of ν ensuring $k_\tau^{SB}(\theta_H) > 0$ and $dk_\tau^{SB}(\theta_H)/d\tau > 0$ differ. For our leading example with cubic abatement costs, we show that these two thresholds coincide.

the Pigouvian tax $\tau^* = \kappa$. Under asymmetric information, however, $\psi'(\cdot) > 0$, which captures the downward distortion in the high-cost type's capital allocation and shifts the optimal tax away from the Pigouvian level. Recall that emissions are decreasing in both the environmental tax and the amount of invested capital, $\partial\hat{q}/\partial\tau < 0$ and $\partial\hat{q}/\partial k < 0$. Moreover, the capital provided to low-cost firms under the second-best contract is increasing in the tax, $dk^{SB}_\tau(\theta_L)/d\tau > 0$. Taken together, these observations imply that if $dk^{SB}_\tau(\theta_H)/d\tau > 0$ – as is the case whenever ν is not too large – then $W'(\tau) > 0$ for all $\tau \leq \kappa$.

Hence, the welfare-maximizing environmental tax exceeds the marginal environmental damage.

Proposition 2. *Suppose the regulator is restricted to a tax-only policy. Then, under asymmetric information, if the share of low-cost firms ν is not too large, the welfare-maximizing environmental tax strictly exceeds the Pigouvian level, $\tau^{**} > \kappa = \tau^*$. The optimal tax is implicitly characterized by*

$$\tau^{**} = \kappa + \frac{\frac{dk^{SB}_{\tau^{**}}(\theta_H)}{d\tau}\left[\frac{\theta_H}{(k^{SB}_{\tau^{**}}(\theta_H))^2}C(\Delta^H_H) - \rho\right]}{\frac{\nu}{1-\nu}\left(\frac{k^{SB}_{\tau^{**}}(\theta_L)}{\theta_L C''\big(\Delta^L_L\big)} + \frac{C'\big(\Delta^L_L\big)}{k^{SB}_{\tau^{**}}(\theta_L)C''\big(\Delta^L_L\big)}\frac{dk^{SB}_{\tau^{**}}(\theta_L)}{d\tau}\right) + \left(\frac{k^{SB}_{\tau^{**}}(\theta_H)}{\theta_H C''\big(\Delta^H_H\big)} + \frac{C'\big(\Delta^H_H\big)}{k^{SB}_{\tau^{**}}(\theta_H)C''\big(\Delta^H_H\big)}\frac{dk^{SB}_{\tau^{**}}(\theta_H)}{d\tau}\right)}, \tag{32}$$

where $\Delta^j_i = q^B - \hat{q}(\tau^{**}k^{SB}_{\tau^{**}}(\theta_j)/\theta_i)$.

It can readily be shown that under the optimal tax-only policy low-cost firms receive too much capital and therefore abate more than under the welfare-optimal allocation, while high-cost firms receive too little capital and consequently abate less than is welfare optimal. Since the optimal tax exceeds the marginal environmental damage, the resulting abatement levels of both types are too high from a welfare perspective, given the capital they receive. The intuition behind Proposition 2 is straightforward. Increasing the environmental tax slightly above the Pigouvian level distorts the capital provision and abatement choices of low-cost firms marginally away from their welfare-optimal levels. At the same time, the higher tax partially offsets the screening-induced downward distortion in capital provision to high-cost firms, thereby moving their capital and abatement levels closer to the social optimum. Thus, the optimal tax-only rate τ^{**} balances these opposing effects: it trades off the additional distortions imposed on low-cost firms against the welfare gains from reducing the distortions affecting high-cost firms.

It is important to note that Proposition 2 relies on the assumption that high-cost firms receive strictly positive amounts of capital for all relevant levels of the environmental tax τ. Furthermore, if high-cost firms are not served under the Pigouvian tax $\tau^* = \kappa$, then the Pigouvian tax is optimal – provided that increasing the tax does not induce the bank to start serving high-cost firms.[10]

4.2. **Environmental Tax and Uniform Subsidy.** Now suppose that the regulator can make use of two instruments: an environmental tax $\tau \geq 0$ and a loan subsidy $\sigma \geq 0$. The subsidy σ is paid to the bank for each unit of capital provided to a firm for green investment, that is, for investment in abatement technology. With the subsidy, the bank's effective unit cost of capital becomes $\hat{\rho} = \rho - \sigma$ instead of ρ. The subsidy may be interpreted as a reduced refinancing rate offered by the central bank for green loans extended by commercial banks.

We assume that the regulator has deep pockets and that there are no shadow costs of public funds. Under these assumptions, the subsidy itself is a welfare-neutral transfer between the regulator (government) and the bank. However, by reducing the bank's capital costs, the subsidy affects the amount of capital supplied, which in turn influences firms' abatement decisions.

If ν is not too large, the second-best loan contracts continue to be characterized by the first-order conditions (20) and (21), with ρ replaced by $\hat{\rho}$. We denote the resulting capital levels by $k^{SB}_{\tau,\sigma}(\theta_i)$, $i \in \{L, H\}$. Since these second-best capital levels are decreasing in the bank's effective cost of capital, both types receive more capital the higher the subsidy:

$$\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial \sigma} > 0 \qquad \text{and} \qquad \frac{\partial k^{SB}_{\tau,\sigma}(\theta_L)}{\partial \sigma} > 0. \tag{33}$$

The regulator maximizes joint surplus – i.e. welfare – which now depends on both policy instruments, $W(\tau, \sigma)$. In general, the joint use of an environmental tax and a credit subsidy leads to interacting effects on equilibrium capital allocation, preventing a precise characterization of the optimal policy mix in the general case. However, the following proposition provides a condition under which Pigouvian taxation is optimal.

Proposition 3. *Suppose the bank's optimization problem has a unique maximizer and both types receive strictly positive amounts of capital. Suppose furthermore that welfare $W(\tau, \sigma)$ exhibits a unique interior stationary point and that point is a maximizer. Then, a regulator who can make use of an environmental tax and a subsidy for green credits optimally sets the environmental tax equal to the Pigouvian level, $\tau^* = \kappa$, and chooses the optimal uniform subsidy implicitly defined by*

$$\sigma^* = \frac{\nu\, \psi'\big(k^{SB}_{\kappa,\sigma^*}(\theta_H)\big)\, \frac{\partial k^{SB}_{\kappa,\sigma^*}(\theta_H)}{\partial \sigma}}{\nu\, \frac{\partial k^{SB}_{\kappa,\sigma^*}(\theta_L)}{\partial \sigma} + (1-\nu)\, \frac{\partial k^{SB}_{\kappa,\sigma^*}(\theta_H)}{\partial \sigma}} > 0, \tag{34}$$

[10]In our leading example with a cubic abatement cost function, the decision of whether high-cost firms are served is independent of the environmental tax.

if and only if the following condition holds at $(\tau,\sigma)=(\kappa,\sigma^*)$*:*

$$\frac{\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial\tau}}{\nu\frac{\partial k^{SB}_{\tau,\sigma}(\theta_L)}{\partial\tau}+(1-\nu)\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial\tau}}=\frac{\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial\sigma}}{\nu\frac{\partial k^{SB}_{\tau,\sigma}(\theta_L)}{\partial\sigma}+(1-\nu)\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial\sigma}}. \tag{35}$$

Condition (35) is not innocuous. It requires that, at the margin, an increase in the environmental tax and an increase in the subsidy affect the second-best capital allocation in the same "relative" way across types. *A priori*, this proportionality condition need not hold. However, in our leading example, which employs a simple and intuitive cubic abatement cost function, condition (35) is satisfied (and likewise for power specifications of degree larger than two, which we do not analyze separately).

When the condition holds, the policy mix of Pigouvian taxation together with the optimal uniform loan subsidy improves upon the allocation achieved under the optimal tax-only policy. However, no policy mix consisting of an environmental tax and a *uniform* subsidy can implement the welfare-optimal allocation. This limitation is clearly illustrated by the policy mix characterized above: since $\sigma^*>0$, it follows that $k^{SB}_{\kappa,\sigma^*}(\theta_L)>k^*(\theta_L)$, and hence the welfare-optimal outcome is not achieved.

4.3. **Environmental Tax and Tailored Subsidy.** We now show that the Pigouvian tax in combination with *tailored* loan subsidies can implement the welfare-optimal allocation. This tailored-subsidy scheme is best understood as a benchmark and thought experiment, since the information required to implement it may be difficult for the regulator to obtain in practice. We return to this implementation issue after stating the result.

Suppose the bank receives a subsidy σ_i, $i\in\{L,H\}$, for each unit of capital provided to a firm of type θ_i. Accordingly, the second-best loan contracts are obtained by replacing the bank's capital cost ρ in the first-order conditions (20) and (21) with $\rho-\sigma_L$ and $\rho-\sigma_H$, respectively.

Under Pigouvian taxation, low-cost firms already receive the efficient amount of capital. Hence, the optimal tailored subsidy for loans to low-cost firms is zero, $\sigma^*_L=0$. For high-cost firms, the subsidy σ^*_H must be set so as to exactly offset the marginal information-rent cost that an increase in their capital provision imposes on the bank; doing so provides the bank with incentives to supply the efficient amount of capital to high-cost firms as well.

Proposition 4. *Under asymmetric information, the Pigouvian tax* $\tau^*=\kappa$ *together with tailored loan subsidies*

$$\sigma^*_L=0 \quad \textit{and} \quad \sigma^*_H=\frac{\nu}{1-\nu}\,\psi'\big(k^*(\theta_H)\big)$$

implements the efficient allocation. That is, for both cost types the policy induces welfare-optimal capital provision and welfare-optimal abatement decisions.

It can readily be shown that the optimal tailored subsidies bracket the optimal uniform subsidy if Pigouvian taxation is optimal; that is, $\sigma_L^* < \sigma^* < \sigma_H^*$.

These tailored subsidies may appear difficult to implement in practice, as they depend on a firm's cost type, which is private information and not observed by the regulator. However, the regulator may nevertheless be able to implement such a scheme without knowing individual firms' types. In equilibrium, only a fraction $1 - \nu$ of firms – the high-cost firms – receive the subsidy, and these firms can be identified by the fact that they are granted the smaller of the two loan amounts. Thus, the regulator can condition the subsidy on the size of the loan rather than on firms' types, effectively reproducing the tailored subsidy scheme.

4.4. **Example (continued).** We return to our example with the cubic abatement cost function $C(\Delta) = \Delta^3/3$.

Tax-only Policy. For this example it follows immediately from (26) that the amounts of capital received by both types are strictly increasing in the environmental tax τ – provided that $k_\tau^{SB}(\theta_H) > 0$, which holds whenever $\nu < \bar{\nu}$. If $\nu \geq \bar{\nu}$, high-cost firms receive no capital, and the optimal tax-only policy sets the Pigouvian tax, $\tau^* = \kappa$. For $\nu < \bar{\nu}$, the optimal tax-only policy instead prescribes a tax above the Pigouvian benchmark, $\tau^{**} > \kappa = \tau^*$. In this case, the optimal tax is given by

$$\tau^{**} = \kappa + \frac{(1-\nu)(1-\beta)\beta}{2\nu\frac{\theta_H}{\theta_L} + (1-\nu)(1+\beta)\beta}\,\kappa, \tag{36}$$

where β is defined in (27). Observe that $\tau^{**} = \kappa$ in the two knife-edge cases $\beta = 0$ and $\beta = 1$, while for all $\beta \in (0,1)$ we have $\tau^{**} > \kappa$. The parameter β approaches zero when either $\nu \to \bar{\nu}$ (a large share of low-cost firms) or when $\theta_H/\theta_L \to 1/\nu^2$ (large type differences). Conversely, $\beta \to 1$ when either $\nu \to 0$ (almost no low-cost firms) or $\theta_H/\theta_L \to 1$ (types nearly identical). These properties imply that the optimal tax τ^{**} is non-monotonic in both the share of low-cost firms ν and the type ratio θ_H/θ_L. The intuition is straightforward: when there are almost no low-cost firms, or when firms are nearly homogeneous, the screening-induced distortions in capital provision are negligible, so the Pigouvian tax is close to optimal. At the other extreme, when the share of low-cost firms is large or when type differences are substantial – so that high-cost firms receive almost no capital – the tax is chosen primarily to maximize welfare generated by low-cost firms, which again calls for a tax close to the Pigouvian level.

Environmental Tax and Uniform Subsidy. By substituting the reduced capital cost $\rho - \sigma$ for ρ in (26), it follows immediately that capital provision to low-cost firms is strictly increasing in the uniform subsidy σ. Capital provision to high-cost firms is also increasing in σ whenever the bank extends credit to high-cost firms. If the bank optimally does not lend to high-cost firms – that is, if $\nu \geq \bar{\nu}$ – the optimal policy consists of the Pigouvian tax, $\tau^* = \kappa$, and no subsidy, $\sigma^* = 0$. Conversely, if $\nu < \bar{\nu}$ and high-cost firms receive capital under the second-best menu, as this example satisfies condition (35), the regulator optimally sets the Pigouvian tax, $\tau^* = \kappa$, together with a positive uniform subsidy

$$\sigma^* = \frac{\rho(1-\nu)(1-\beta)\beta}{\nu\frac{\theta_H}{\theta_L} + (1-\nu)\beta}. \tag{37}$$

The optimal uniform subsidy is zero, $\sigma^* = 0$, in the two extreme cases $\beta = 0$ and $\beta = 1$, while $\sigma^* > 0$ for all $\beta \in (0, 1)$. When $\beta = 0$, the bank does not lend to high-cost firms even with a subsidy; therefore, it is optimal not to subsidize credit (and indeed $\sigma^* \to 0$ as $\beta \to 0$). When $\beta = 1$, there is no downward distortion in the capital provided to high-cost firms, implying that no subsidy is required. These knife-edge properties mirror those of the optimal tax-only solution: in environments with either negligible informational frictions or extremely severe ones, the policy again converges to its benchmark level. Hence, the optimal uniform subsidy σ^* is non-monotonic in both the share of low-cost firms ν and the type ratio θ_H/θ_L.

Environmental Tax and Tailored Subsidy. To implement the welfare-optimal allocation, the regulator sets the Pigouvian tax, $\tau^* = \kappa$, together with tailored subsidies

$$\sigma_L^* = 0 \quad \text{and} \quad \sigma_H^* = \rho(1-\beta). \tag{38}$$

The bank therefore never receives a subsidy for lending to low-cost firms, while lending to high-cost firms is subsidized by σ_H^*. It is immediate that the optimal uniform subsidy lies strictly between the two tailored subsidies. The tailored subsidy for high-cost firms σ_H^* is strictly decreasing in β. From (26), a smaller value of β corresponds to a larger downward distortion in the capital provided to high-cost firms under the second-best contract. Consequently, a larger share of low-cost firms ν or a larger type ratio θ_H/θ_L (both of which reduce β) require a higher tailored subsidy σ_H^* to offset this distortion. In the limit case where the distortion becomes arbitrarily large, $\beta \to 0$ (but $\beta > 0$), the regulator must make it effectively costless for the bank to lend to high-cost firms, and the required subsidy converges to the full capital cost, $\sigma_H^* \to \rho$.

5. Continuum of Cost Types

In this section, we show that our main findings derived under the assumption of two cost types carry over to settings with more than two types. Specifically, we analyze a version of the model with a continuum of cost types.

5.1. **General Distribution of Cost Types.** Suppose that each firm is characterized by a type $\theta \in [\underline{\theta}, \bar{\theta}] \equiv \Theta$, with $0 < \underline{\theta} < \bar{\theta}$. There is a continuum of firms of measure one in the industry. We assume that types are distributed according to the density $f(\theta) > 0$ and the c.d.f. $F(\theta)$. The abatement cost of a firm of type θ that receives and invests capital $k > 0$ is still given by

$$\frac{\theta}{k} C(\Delta), \tag{39}$$

where the abatement cost function $C(\cdot)$ continues to satisfy Assumptions 1 and 2.

The type θ is a firm's private information. The bank offers a direct revelation mechanism $\langle k(\theta), R(\theta) \rangle_{\theta \in \Theta}$, specifying an amount of capital $k(\theta)$ and a repayment $R(\theta)$ for each type θ. The advantage of a firm of type θ from accepting the contract $(k(\theta), R(\theta))$ designated for it is

$$V(\theta) = \tau q^B + \underbrace{\left\{ -\tau \hat{q}\left(\frac{\tau k(\theta)}{\theta}\right) - \frac{\theta}{k(\theta)} C\left(q^B - \hat{q}\left(\frac{\tau k(\theta)}{\theta}\right)\right)\right\}}_{=u(k(\theta),\theta)} - R(\theta), \tag{40}$$

where the optimal pollution level is defined by (10). We call $u(k, \theta)$ the "utility" that type θ derives from obtaining capital k. As can be shown, the utility is decreasing in the type; i.e., firms of a lower cost type realize a larger increase in profits from a given contract. The benefit from accepting a contract, measured by the increase in profit, is higher the larger the amount of capital provided. Finally, firms of a lower cost type benefit more from an increase in the amount of capital provided. This ensures that a capital schedule $k(\theta)$ that is decreasing in the type θ can screen types.

Second-best Optimal Capital Schedule. The bank solves the following maximization problem:

$$\max_{\langle k(\theta), R(\theta) \rangle_{\theta \in \Theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left[R(\theta) - \rho k(\theta)\right] f(\theta)\, \mathrm{d}\theta \tag{41}$$

subject to: for all $\theta \in \Theta$,

$$V(\theta) \geq 0 \tag{PC}$$

$$V(\theta) \geq V(\hat{\theta}) + u(k(\hat{\theta}), \theta) - u(k(\hat{\theta}), \hat{\theta}) \quad \forall\, \hat{\theta} \in \Theta. \tag{IC}$$

The bank maximizes its profit, final repayments minus capital costs, subject to the participation constraint (PC) and the incentive compatibility constraint (IC). Each type must receive a non-negative net gain from accepting the contract designated for this type, as required by (PC). Moreover, each cost type must prefer to announce its type truthfully; that is, each type must weakly prefer the designated contract to all other contracts, as required by (IC).

The screening problem of the bank can be simplified using the usual steps. It can be rewritten as a problem of choosing only a capital schedule. If this capital schedule is strictly decreasing, so that the monotonicity constraint is slack, the second-best optimal capital schedule is obtained by point-wise optimization.[11] Let $k_\tau^{FB}(\theta)$ denote the full-information capital level for type θ under environmental tax τ, which is still characterized by (15).

Lemma 4. *Suppose that the second-best optimal capital schedule, $k_\tau^{SB}(\theta)$, is strictly decreasing and satisfies $k_\tau^{SB}(\bar{\theta}) > 0$. Then, the second-best optimal capital schedule solves*

$$(42) \quad \frac{\theta}{k^2} C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta}\right)\right) + \frac{F(\theta)}{f(\theta)} \frac{1}{k^2}\left[C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta}\right)\right) - \frac{\left[C'\left(q^B - \hat{q}\left(\frac{\tau k}{\theta}\right)\right)\right]^2}{C''\left(q^B - \hat{q}\left(\frac{\tau k}{\theta}\right)\right)}\right] = \rho.$$

It features no distortion for the most efficient type, $k_\tau^{SB}(\underline{\theta}) = k_\tau^{FB}(\underline{\theta})$, and a downward distortion for all other types, $k_\tau^{SB}(\theta) < k_\tau^{FB}(\theta)$ for all $\theta > \underline{\theta}$.

It is important to note that the term in square brackets in equation (42) is negative by Assumption 2. Therefore, the first-order condition (42) can be satisfied only at capital levels below the full-information capital level. If the environmental tax equals marginal environmental damage, $\tau = \kappa$, all firms choose the optimal abatement level conditional on the capital they receive. However, only the most efficient firm type, $\theta = \underline{\theta}$, receives the optimal amount of capital, whereas all other firms receive too little capital. These firms invest too little, abate too little, and pollute too much from a welfare point of view.

Welfare Analysis – Tax-only Policy. For the policy analysis of the general case with a continuum of cost types, we focus on the tax-only policy. In other words, the regulator chooses an environmental tax τ in order to maximize aggregate social surplus. Aggregate social surplus, or welfare, as a function of the environmental tax is given by

$$(43) \quad W(\tau) = \int_{\underline{\theta}}^{\bar{\theta}} \Bigg\{ \Pi - \frac{\theta}{k_\tau^{SB}(\theta)} C\left(q^B - \hat{q}\left(\frac{\tau k_\tau^{SB}(\theta)}{\theta}\right)\right) \\ - \kappa \hat{q}\left(\frac{\tau k_\tau^{SB}(\theta)}{\theta}\right) - \rho k_\tau^{SB}(\theta) \Bigg\} f(\theta) \mathrm{d}\theta$$

[11] Sufficient conditions under which the monotonicity constraint is slack are provided in Appendix B.2.

An increase in the environmental tax τ leads to the following change in welfare:

$$W'(\tau) = \int_{\underline{\theta}}^{\bar{\theta}} \Bigg\{ -(\tau-\kappa)\frac{1}{\theta C''\left(q^B - \hat{q}(\frac{\tau k_\tau^{SB}(\theta)}{\theta})\right)}\left[k_\tau^{SB}(\theta) + \tau\frac{dk_\tau^{SB}}{d\tau}\right] + \left[\frac{\theta}{[k_\tau^{SB}(\theta)]^2}C\left(q^B - \hat{q}\left(\frac{\tau k_\tau^{SB}(\theta)}{\theta}\right)\right) - \rho\right]\frac{dk_\tau^{SB}}{d\tau}\Bigg\} f(\theta)\mathrm{d}\theta \tag{44}$$

Roughly speaking, the first term captures the direct effect through firms' abatement and, hence, emission adjustments induced by the change in the environmental tax. The second term captures the indirect effect through the induced change in capital provision. While the first term vanishes under the Pigouvian tax $\tau^* = \kappa$, this is not the case for the second term. The regulator therefore trades off a slight distortion of firms' abatement decisions against the benefit of bringing capital levels closer to their efficient levels.

As in the two-type case, it is important how the second-best capital schedule reacts to a change in the environmental tax. In the intuitive case in which capital provision is increasing in the environmental tax, the regulator has an incentive to choose a tax above the Pigouvian level. Recall that the term in square brackets in the second line of (44) is strictly positive for all types except $\underline{\theta}$ by the definition of the second-best capital schedule.

Proposition 5. *Suppose the regulator is restricted to a tax-only policy. If the second-best optimal capital schedule $k_\tau^{SB}(\theta)$ is characterized by* (42) *and satisfies $dk_\tau^{SB}(\theta)/d\tau > 0$ for all $\theta \in \Theta$, then the welfare-maximizing environmental tax strictly exceeds the Pigouvian level, $\tau^{**} > \kappa = \tau^*$.*

Proposition 5 establishes that Proposition 2 is not an artifact of the two-type case, but extends to the case of a continuum of types. Both propositions rely on the assumption that the second-best capital schedule is increasing in the environmental tax. While in the two-type case sufficient conditions for this assumption can be found relatively easily, this is less straightforward in the case of a continuum of types. In Appendix B.2, we provide sufficient conditions under which this property holds. We next show that the property is satisfied in an extension of our leading example.

5.2. **Example (Adjusted): Cubic Abatement Cost Function & Uniform Type Distribution.** To illustrate the preceding continuum result, we return to the cubic specification, $C(\Delta) = \Delta^3/3$, and assume that types are uniformly distributed on $[\underline{\theta}, \bar{\theta}]$. Thus, $F(\theta) = (\theta - \underline{\theta})/(\bar{\theta} - \underline{\theta})$ and $f(\theta) = 1/(\bar{\theta} - \underline{\theta})$. Welfare-optimal and full-information capital levels are still characterized by (23) and (25), respectively, while capital provision under asymmetric information is now given by

$$k_\tau^{SB}(\theta) = \frac{\tau^3}{36\,\rho^2\,\theta}\left(1 + \frac{\underline{\theta}}{\theta}\right)^2. \tag{45}$$

Capital provision is strictly positive for every type, a property that follows from the uniform type distribution.[12] The schedule coincides with the full-information benchmark only for the most efficient type, $\theta = \underline{\theta}$, and lies strictly below it for all higher-cost types, $\theta > \underline{\theta}$. It is monotone in the required direction, i.e., decreasing in the cost type θ, and capital provision is increasing in the environmental tax τ.

We first consider the tax-only policy. Since capital provision is increasing in the environmental tax for all types in this example, Proposition 5 implies that the optimal tax-only policy sets the environmental tax above the Pigouvian level. Defining the relative spread of the type distribution $h := \bar{\theta}/\underline{\theta} > 1$ as a measure of type heterogeneity and

$$\omega(h) := \frac{2\ln h - 1 + h^{-2}}{4\left(\ln h + 1 - h^{-1}\right)},$$

with $\omega(h) \in (0, 1/2)$, $\omega'(h) > 0$, and $\lim_{h\to 1} \omega(h) = 0$, the optimal tax is given by

$$\tau^{**} = \kappa + \frac{\omega(h)}{2 - \omega(h)}\kappa. \tag{46}$$

It follows immediately that $\tau^{**} > \kappa$. Moreover, the upward adjustment relative to the Pigouvian level, $\tau^{**} - \kappa$, is strictly increasing in type heterogeneity h. As type heterogeneity vanishes, $h \to 1$, the optimal tax converges to the Pigouvian level, $\tau^{**} \to \kappa$.

The tractability of the cubic-uniform example also allows us to characterize the optimal policy when the environmental tax is combined with a uniform subsidy. The result closely parallels the two-type example. The optimal policy mix again consists of the Pigouvian tax, $\tau^* = \kappa$, and a uniform subsidy,

$$\sigma^* = \rho\omega(h), \tag{47}$$

which is strictly positive and strictly increasing in the relative spread of the type distribution h. As type heterogeneity vanishes, we have $\sigma^* \to 0$.[13]

[12] For cubic abatement costs and a general type distribution, the relaxed point-wise solution assigns strictly positive capital to type θ if and only if $2\theta - F(\theta)/f(\theta) > 0$. Whenever the monotonicity constraint is slack, capital provision is then given by

$$k_\tau^{SB}(\theta) = \frac{\tau^3}{36\rho^2\theta^3}\left(2\theta - \frac{F(\theta)}{f(\theta)}\right)^2.$$

[13] Unlike in the two-type example, both the optimal tax-only policy τ^{**} and the optimal uniform subsidy σ^* increase monotonically with heterogeneity in the cubic-uniform continuum example. This difference is not driven by the continuum of types per se. As the support widens under the uniform distribution, additional higher-cost types remain served, and their capital provision remains bounded away from zero relative to the full-information benchmark. Hence, the corrective lending margin does not disappear. In the two-type example, by contrast, the high-cost type effectively drops out as $\beta \to 0$, weakening the corrective motive. Other type distributions may therefore yield non-monotonic policy responses.

6. Conclusion

This paper analyzes optimal environmental policy when firms must finance abatement investment through bank borrowing and credit is allocated under adverse selection. Firms privately know the effectiveness of their abatement investment, and a monopolistic bank screens them via a menu of loan contracts. The resulting screening distortion reduces capital provision to high-cost firms below the efficient level: they receive too little capital and therefore abate too little, leading to excessive emissions.

Our analysis delivers three main insights. First, when the regulator is restricted to an emissions tax, the welfare-maximizing tax exceeds the Pigouvian benchmark whenever high-cost firms are served. A higher tax mitigates the downward capital distortion by expanding lending to high-cost firms, even though it distorts abatement decisions for given capital levels. This result hinges on the fact that, in our model, capital provision and abatement investment increase with the emissions tax. If instead a higher tax tightened firms' financing constraints – for example, because more resources are absorbed by tax payments and fewer remain available for investment – abatement investment could decline with the tax. In such settings, the optimal tax may lie below marginal environmental damages, that is, below the Pigouvian level. Second, when the regulator can complement taxation with a uniform subsidy for green lending, it is optimal – under a broad class of cost functions that include our leading example – to return to Pigouvian taxation and use the subsidy to mitigate credit-market distortions. However, a uniform subsidy cannot implement the first-best allocation because it also expands lending to firms that are already efficiently financed. Third, efficiency can be restored by combining the Pigouvian tax with targeted loan subsidies that apply only to the contract chosen by high-cost firms. In line with the Tinbergen rule, the tax corrects the environmental externality, while the targeted subsidy addresses the type-specific distortion in capital provision. In practice, however, such targeting may be difficult because the relevant firm characteristics are private information. Conditioning support on observable contract characteristics, such as loan size, may approximate the targeted subsidy scheme, but its feasibility becomes more limited as firm heterogeneity becomes richer, for instance because there are more types.

These findings highlight that the optimal design of carbon pricing cannot be separated from the functioning of green credit markets. When abatement depends on external finance, emissions taxes affect not only firms' marginal abatement incentives but also equilibrium credit supply. Broad credit support measures may improve welfare, yet they inevitably generate windfalls and overinvestment among unconstrained firms. Policies that focus support on the distorted margin, by contrast, address the source of inefficiency more directly and limit unintended spillovers.

Several extensions merit further investigation. In practice, fiscal resources, political constraints, and administrative frictions may limit the feasibility or precision of targeted credit support. Moreover, carbon pricing itself is often politically constrained. An important avenue for future research is therefore to study optimal policy design when both instruments are imperfect or quantitatively limited. More generally, future work could examine environments in which environmental taxes tighten rather than relax financing constraints. In such cases, the interaction between carbon pricing and credit frictions may call for lower emissions taxes than in the mechanism studied here.

APPENDIX A. PROOFS

Proof of Lemma 1. Under full information, the bank can distinguish between the two types and thus chooses the contract for each type such that the participation constraint (PC_i) holds with equality, leading to (14). From the first-order condition

$$\frac{\partial B_\theta}{\partial k(\theta)}(k(\theta)) = \frac{d\hat{q}}{dz}\frac{dz}{dk(\theta)}\underbrace{\left[\frac{\theta}{k(\theta)}C'(q^B - \hat{q}(z)) - \tau\right]}_{=0} + \frac{\theta}{k(\theta)^2}C(q^B - \hat{q}(z)) - \rho \overset{!}{=} 0, \tag{A.1}$$

expression (15) follows directly. For brevity, we write everywhere in the appendix Δ instead of $\hat{\Delta}$. Because

$$\frac{\partial^2 B_\theta}{\partial k(\theta)^2}(k(\theta)) = \frac{\theta}{k(\theta)^3}\left[\frac{C'(\Delta)^2}{C''(\Delta)} - 2C(\Delta)\right] < 0 \tag{A.2}$$

by Assumption 2, the bank's profit from each type is globally strictly concave. The bank's type-specific profit is not indefinitely increasing in capital provision ($k(\theta) \to +\infty$ cannot be profit-maximizing). Therefore, (15) implicitly characterizes for both types θ_i, $i \in \{L, H\}$, the full-information profit-maximizing capital levels $k_\tau^{FB}(\theta_i)$. □

Proof of Proposition 1. This result follows immediately from comparing (15) to (5) and (10) to (6). □

Proof of Lemma 2. From (19), using (10), the first-order condition (20) is readily obtained. For high-cost firms, the first-order condition is

$$\frac{\partial B}{\partial k_H}(k_L, k_H) = -\nu\psi'(k_H) + (1-\nu)\left[\frac{\theta_H}{k_H^2}C\left(q^B - \hat{q}\left(\frac{\tau k_H}{\theta_H}\right)\right) - \rho\right] \overset{!}{=} 0. \tag{A.3}$$

To observe more immediately that if ν is small enough (and because we assume that the bank lends to high-cost firms in the full-information benchmark), the bank indeed serves both types under asymmetric information, the first-order condition can be written as

$$\frac{1}{(1-\nu)k_H^2}\left[\theta_H C(\Delta_H^H) - \nu\theta_L C(\Delta_L^H)\right] = \rho, \tag{A.4}$$

using

$$\Delta_i^j := q^B - \hat{q}\left(\frac{\tau k_\tau^{SB}(\theta_j)}{\theta_i}\right), \qquad i, j \in \{L, H\}.$$

From the above, (21) follows immediately.

Comparing (20) to (15) yields $k_\tau^{FB}(\theta_L) = k_\tau^{SB}(\theta_L)$. $k_\tau^{FB}(\theta_H) > k_\tau^{SB}(\theta_H)$ follows from

$$\psi'(k) = \frac{1}{k^2}\left[\theta_L C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta_L}\right)\right) - \theta_H C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta_H}\right)\right)\right] > 0, \tag{A.5}$$

because $a(\theta) := \theta C(\Delta)$ is monotonically decreasing in θ by Assumption 2,

$$a'(\theta) = C(\Delta) - \frac{C'(\Delta)^2}{C''(\Delta)} < 0, \tag{A.6}$$

and $g(k) := \theta C(\Delta)/k^2$ is strictly decreasing in k,

$$g'(k) = \frac{\theta}{k^3}\left[\frac{C'(\Delta)^2}{C''(\Delta)} - 2C(\Delta)\right] < 0, \tag{A.7}$$

again by Assumption 2.

The Hessian H for the bank's objective is

$$H = \begin{pmatrix} \frac{\partial^2 B}{\partial k_L^2} & 0 \\ 0 & \frac{\partial^2 B}{\partial k_H^2} \end{pmatrix},$$

with

$$\frac{\partial^2 B}{\partial k_L^2} < 0 \tag{A.8}$$

by Assumption 2 (see above). Moreover, for an interior stationary point at which the bank lends to both types, a sufficient second-order condition for a strict local maximum in k_H is

$$\frac{\partial^2 B}{\partial k_H^2}\left(k_\tau^{SB}(\theta_L), k_\tau^{SB}(\theta_H)\right) < 0. \tag{A.9}$$

This second-order condition for k_H can also be written as

$$\theta_H\left(2C(\Delta_H^H) - \frac{[C'(\Delta_H^H)]^2}{C''(\Delta_H^H)}\right) > \nu\theta_L\left(2C(\Delta_L^H) - \frac{[C'(\Delta_L^H)]^2}{C''(\Delta_L^H)}\right), \tag{A.10}$$

i.e., if ν is small enough (albeit characterized by an endogenous condition), the second-order condition holds. Together, this implies that the first-order conditions (20) and (21) characterize the second-best profit-maximizing capital levels under asymmetric information, provided that ν is not too large.

Note that if the bank does not lend to high-cost firms, $k_\tau^{FB}(\theta_H) > k_\tau^{SB}(\theta_H) = 0$ still holds, given that the bank lends to both types under full information. □

Proof of Corollary 1. Under Pigouvian taxation $\tau^* = \kappa$, Proposition 1 and Lemma 2 imply $k_\kappa^{SB}(\theta_L) = k_L^*$ and $k_\kappa^{SB}(\theta_H) < k_H^*$. Since $\hat{q}(\tau k/\theta)$ is decreasing in k, it follows that $\hat{q}\left(\frac{\kappa k_\kappa^{SB}(\theta_L)}{\theta_L}\right) = q_L^*$ and $\hat{q}\left(\frac{\kappa k_\kappa^{SB}(\theta_H)}{\theta_H}\right) > q_H^*$. □

Proof of Lemma 3. Consider first the low-cost firms. As already established, $\frac{\partial^2 B}{\partial k_L^2} < 0$. We again write $\Delta_i^j = q^B - \hat{q}(\tau k_\tau^{SB}(\theta_j)/\theta_i)$, $i, j \in \{L, H\}$. Because

$$\frac{\partial^2 B}{\partial k_L \partial \tau} = \nu \frac{1}{k_L} \frac{C'(\Delta_L^L)}{C''(\Delta_L^L)} > 0, \tag{A.11}$$

we obtain from the Implicit Function Theorem (IFT)

$$\frac{dk_\tau^{SB}(\theta_L)}{d\tau} > 0.$$

Consider now the high-cost firms. Recall from above, that if ν is small enough, lending to high-cost firms occurs and we have $\frac{\partial^2 B}{\partial k_H^2} < 0$. Substituting for $\psi'(k_H)$ we obtain

$$\frac{\partial^2 B}{\partial k_H \partial \tau} = \frac{1}{k_H} \left[\frac{C'(\Delta_H^H)}{C''(\Delta_H^H)} - \nu \frac{C'(\Delta_L^H)}{C''(\Delta_L^H)} \right]. \tag{A.12}$$

Thus, if ν is not too large, $\frac{\partial^2 B}{\partial k_H \partial \tau} > 0$, and

$$\frac{dk_\tau^{SB}(\theta_H)}{d\tau} > 0$$

follows. □

Proof of Proposition 2. By the argument outlined in the main text, it is obvious from (31) that the optimal tax strictly exceeds the Pigouvian level, $\tau^{**} > \kappa$. Expression (32) follows from (31) after inserting the relevant expressions and simplifying the first-order condition $W'(\tau) \stackrel{!}{=} 0$. It remains to show that (32) indeed implicitly characterizes the optimal tax τ^{**}. By right-continuity of $W(\cdot)$ at $\tau = 0$ (which follows because the induced capital and abatement levels vanish as $\tau \to 0$) and since $W'(\tau) > 0$ for all sufficiently small $\tau > 0$ (see (31)), $\tau = 0$ cannot be welfare-maximizing. Furthermore, welfare is not indefinitely increasing in τ, because $W(\tau)$ would be decreasing at the latest after that τ that induces $k_\tau^{SB}(\theta_H) = k_H^*$. Moreover, if capital provision to the high-cost type faces an upper bound below k_H^*, then $\frac{dk_\tau^{SB}(\theta_H)}{d\tau}$ would go to zero, again yielding from (31) that welfare would decrease for a large enough τ. □

Proof of Proposition 3. Welfare is given by

$$\begin{aligned}
\text{(A.13)}\quad W(\tau,\sigma) = \\
\nu\left[\Pi - \frac{\theta_L}{k^{SB}_{\tau,\sigma}(\theta_L)}\,C\left(q^B - \hat{q}\left(\frac{\tau\,k^{SB}_{\tau,\sigma}(\theta_L)}{\theta_L}\right)\right) - \kappa\,\hat{q}\left(\frac{\tau\,k^{SB}_{\tau,\sigma}(\theta_L)}{\theta_L}\right) - \rho\,k^{SB}_{\tau,\sigma}(\theta_L)\right] \\
+(1-\nu)\left[\Pi - \frac{\theta_H}{k^{SB}_{\tau,\sigma}(\theta_H)}\,C\left(q^B - \hat{q}\left(\frac{\tau\,k^{SB}_{\tau,\sigma}(\theta_H)}{\theta_H}\right)\right) - \kappa\,\hat{q}\left(\frac{\tau\,k^{SB}_{\tau,\sigma}(\theta_H)}{\theta_H}\right) - \rho\,k^{SB}_{\tau,\sigma}(\theta_H)\right].
\end{aligned}$$

The derivative w.r.t. τ is:

$$\begin{aligned}
\text{(A.14)}\quad \frac{\partial W}{\partial \tau} &= (\tau-\kappa)\left\{\nu\left[\frac{\partial \hat{q}_L}{\partial \tau} + \frac{\partial \hat{q}_L}{\partial k}\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_L)}{\partial \tau}\right] + (1-\nu)\left[\frac{\partial \hat{q}_H}{\partial \tau} + \frac{\partial \hat{q}_H}{\partial k}\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial \tau}\right]\right\} \\
&\quad + \nu\,\psi'\big(k^{SB}_{\tau,\sigma}(\theta_H)\big)\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial \tau} - \sigma\Big[\nu\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_L)}{\partial \tau} + (1-\nu)\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial \tau}\Big].
\end{aligned}$$

The derivative w.r.t. σ is:

$$\begin{aligned}
\text{(A.15)}\quad \frac{\partial W}{\partial \sigma} &= (\tau-\kappa)\left\{\nu\,\frac{\partial \hat{q}_L}{\partial k}\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_L)}{\partial \sigma} + (1-\nu)\,\frac{\partial \hat{q}_H}{\partial k}\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial \sigma}\right\} \\
&\quad + \nu\,\psi'\big(k^{SB}_{\tau,\sigma}(\theta_H)\big)\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial \sigma} - \sigma\Big[\nu\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_L)}{\partial \sigma} + (1-\nu)\,\frac{\partial k^{SB}_{\tau,\sigma}(\theta_H)}{\partial \sigma}\Big].
\end{aligned}$$

If $W(\tau,\sigma)$ has a unique interior stationary point and that point is a maximizer, it follows immediately that condition (35) at $(\tau,\sigma) = (\kappa,\sigma^*)$ is a necessary and sufficient condition for Pigouvian taxation, $\tau = \kappa$, to be optimal. Plugging $\tau = \kappa$ into the first-order condition $\frac{\partial W}{\partial \sigma} \overset{!}{=} 0$ then yields the optimal uniform subsidy σ^* implicitly characterized by (34). □

Proof of Proposition 4. The result follows immediately from the argument in the main text. □

Proof of Lemma 4. The proof proceeds in four steps. First, we derive useful properties of the function $u(k,\theta)$. Second, we simplify the incentive compatibility constraint. Third, we rewrite the bank's optimization problem as a problem of choosing only a capital schedule. Finally, we establish the remaining parts of the lemma.

First, we show certain properties of the "utility function" $u(k,\theta)$. The partial derivatives with respect to θ and k are

$$\text{(A.16)}\qquad \frac{\partial u(k,\theta)}{\partial \theta} = -\frac{1}{k}C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta}\right)\right) < 0,$$

$$\text{(A.17)}\qquad \text{and}\quad \frac{\partial u(k,\theta)}{\partial k} = \frac{\theta}{k^2}C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta}\right)\right) > 0,$$

respectively. The cross derivative can be written as

$$\frac{\partial^2 u(k,\theta)}{\partial k \partial \theta} = \frac{1}{k^2}\left[C(q^B - \hat{q}) - \frac{[C'(q^B - \hat{q})]^2}{C''(q^B - \hat{q})}\right] < 0. \tag{A.18}$$

The cross derivative is negative because, by Assumption 2, $C'/C > C''/C'$. Thus, the single-crossing property is satisfied.

Incentive-Compatible Mechanisms: Incentive compatibility is satisfied if, for any $\theta \in \Theta$,

$$V(\theta) \geq \tau q^B - \tau \hat{q}\left(\frac{\tau k(\hat{\theta})}{\theta}\right) - \frac{\theta}{k(\hat{\theta})} C\left(q^B - \hat{q}\left(\frac{\tau k(\hat{\theta})}{\theta}\right)\right) - R(\hat{\theta}) \quad \forall \hat{\theta} \in \Theta. \tag{A.19}$$

Using the defined "utility function", this expression can be written as

$$V(\theta) \geq V(\hat{\theta}) + u(k(\hat{\theta}),\theta) - u(k(\hat{\theta}),\hat{\theta}) \quad \forall \hat{\theta} \in \Theta. \tag{A.20}$$

Now consider two types, θ_1 and θ_2, with $\theta_2 > \theta_1$. Neither type has an incentive to claim to be the other type if and only if

$$-\int_{\theta_1}^{\theta_2} \frac{\partial u(k(\theta_1),\theta)}{\partial \theta} \mathrm{d}\theta \geq V(\theta_1) - V(\theta_2) \geq -\int_{\theta_1}^{\theta_2} \frac{\partial u(k(\theta_2),\theta)}{\partial \theta} \mathrm{d}\theta. \tag{A.21}$$

We can draw three important observations from (A.21).

(i) $V(\theta_1) \geq V(\theta_2)$, with strict inequality if $k(\theta_2) > 0$, because then $\partial u/\partial \theta < 0$.

(ii) Ignoring the term in the middle, (A.21) requires that

$$-\int_{\theta_1}^{\theta_2} \int_{k(\theta_2)}^{k(\theta_1)} \frac{\partial^2 u(k,\theta)}{\partial \theta \partial k} \mathrm{d}k \mathrm{d}\theta \geq 0. \tag{A.22}$$

Since $\partial^2 u/\partial k \partial \theta < 0$, the above condition holds if and only if $k(\theta_1) \geq k(\theta_2)$. Put verbally, any incentive-compatible capital schedule is non-increasing.

(iii) Finally, for $\theta_1 \to \theta_2$, we see that $V(\cdot)$ is absolutely continuous with respect to Lebesgue measure and thus almost everywhere differentiable; i.e., $V'(\theta)$ exists (a.e.).

In order to derive $V'(\theta)$, note that differentiability of $V(\cdot)$ a.e. implies differentiability of $R(\theta)$ a.e. Let $v(\tilde{\theta},\theta)$ be the surplus obtained by type θ who claims to be of type $\tilde{\theta}$. Formally,

$$v(\tilde{\theta},\theta) := \tau\left[q^B - \hat{q}\left(\frac{\tau k(\tilde{\theta})}{\theta}\right)\right] - \frac{\theta}{k(\tilde{\theta})} C\left(q^B - \hat{q}\left(\frac{\tau k(\tilde{\theta})}{\theta}\right)\right) - R(\tilde{\theta}). \tag{A.23}$$

Note that, by definition, $V(\theta) = v(\theta,\theta)$. The first-order condition $\partial v(\theta,\theta)/\partial \tilde{\theta} = 0$, i.e., local incentive compatibility, is satisfied if and only if

$$R'(\theta) = \frac{\theta}{[k(\theta)]^2} C\left(q^B - \hat{q}\left(\frac{\tau k(\theta)}{\theta}\right)\right) \frac{dk(\theta)}{d\theta}. \tag{A.24}$$

The derivative of the surplus $V(\theta)$ is therefore given by the envelope condition

$$V'(\theta) = \underbrace{-R'(\theta) + \frac{\partial u(k(\theta),\theta)}{\partial k}\frac{dk(\theta)}{d\theta}}_{=0} + \frac{\partial u(k(\theta),\theta)}{\partial \theta}$$

$$= -\frac{1}{k(\theta)} C\left(q^B - \hat{q}\left(\frac{\tau k(\theta)}{\theta}\right)\right) < 0. \tag{A.25}$$

The absolute continuity of $V(\cdot)$ allows us to write

$$V(\theta) = V(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} \frac{\partial u(k(z),z)}{\partial z} \mathrm{d}z. \tag{A.26}$$

Bank's Optimization Problem and the Optimal Mechanism: Recall that the gross repayment is given by

$$R(\theta) = \tau q^B + u(k(\theta),\theta) - V(\theta)$$

$$= \tau q^B + u(k(\theta),\theta) - V(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \frac{\partial u(k(z),z)}{\partial z} \mathrm{d}z. \tag{A.27}$$

By the single-crossing property, any non-increasing capital schedule together with an absolutely continuous rent schedule satisfying the envelope condition almost everywhere is globally incentive compatible. Hence, the bank's original screening problem can equivalently be rewritten as a problem of choosing the capital schedule $k(\theta)$ and the boundary rent $V(\bar{\theta})$:

$$\max_{\langle k(\theta)\rangle_{\theta\in\Theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left\{\tau q^B + u(k(\theta),\theta) - V(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \frac{\partial u(k(z),z)}{\partial z}\mathrm{d}z - \rho k(\theta)\right\} f(\theta)\mathrm{d}\theta \tag{A.28}$$

subject to

(PC) $\quad V(\bar{\theta}) \geq 0$

(MON) $\quad k(\theta)$ non-increasing

It is optimal for the bank not to leave a rent to the least efficient type, i.e., to set $V(\bar{\theta}) = 0$. We presume that constraint (MON) is non-binding.

Using integration by parts allows us to rewrite the double integral in the following form:

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\bar{\theta}} \frac{\partial u(k(z),z)}{\partial z}\mathrm{d}z \; f(\theta)\mathrm{d}\theta = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial u(k(\theta),\theta)}{\partial \theta} F(\theta)\mathrm{d}\theta. \tag{A.29}$$

Thus, the bank's objective function is

$$B = \int_{\underline{\theta}}^{\bar{\theta}} \left\{\tau q^B + u(k(\theta),\theta) + \frac{\partial u(k(\theta),\theta)}{\partial \theta}\frac{F(\theta)}{f(\theta)} - \rho k(\theta)\right\} f(\theta)\mathrm{d}\theta. \tag{A.30}$$

If constraint (MON) is slack, point-wise maximization techniques can be applied to solve the bank's maximization problem. In this regard, let

$$b(k|\theta) = \tau q^B + u(k,\theta) + \frac{\partial u(k,\theta)}{\partial \theta}\frac{F(\theta)}{f(\theta)} - \rho k. \tag{A.31}$$

The first-order condition for optimality is

$$\frac{\partial u(k,\theta)}{\partial k} + \frac{F(\theta)}{f(\theta)}\frac{\partial^2 u(k,\theta)}{\partial \theta \partial k} = \rho. \tag{A.32}$$

Replacing the derivatives of $u(k,\theta)$ in (A.32) with their explicit expressions yields (42), as stated in the lemma.

Further results: For $\underline{\theta}$ it holds that $F(\underline{\theta}) = 0$, and thus the second-best optimal capital solves

$$\frac{\underline{\theta}}{k^2} C\left(q^B - \hat{q}\left(\frac{\tau k}{\underline{\theta}}\right)\right) = \rho. \tag{A.33}$$

Hence, $k_\tau^{SB}(\underline{\theta}) = k_\tau^{FB}(\underline{\theta})$.

For $\theta > \underline{\theta}$, we have $F(\theta) > 0$. Moreover, by Assumption 2, the term in square brackets in (42) is negative. Hence, the first-order condition can be satisfied only for capital levels such that

$$\frac{\theta}{k^2} C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta}\right)\right) > \rho. \tag{A.34}$$

If the left-hand side of (A.34) is decreasing in k, then the $k(\theta)$ that solves (42) has to be strictly smaller than $k_\tau^{FB}(\theta)$. The derivative of the left-hand side of (A.34) with respect to k is

$$\begin{aligned} &-\frac{2\theta}{k^3} C(q^B - \hat{q}) + \frac{\theta}{k^2}\frac{C'(q^B-\hat{q})}{C''(q^B-\hat{q})}\frac{\tau}{\theta} \\ &= \frac{\theta}{k^3}\left\{ -2C(q^B-\hat{q}) + \frac{\left[C'(q^B-\hat{q})\right]^2}{C''(q^B-\hat{q})}\right\} < 0. \end{aligned} \tag{A.35}$$

The term is strictly negative by Assumption 2. Hence, $k_\tau^{SB}(\theta) < k_\tau^{FB}(\theta)$ for all $\theta > \underline{\theta}$, which completes the proof. $\square$

Proof of Proposition 5. Taking the derivative of (43) with respect to τ yields

$$\begin{aligned} W'(\tau) = \int_{\underline{\theta}}^{\bar{\theta}} \Bigg\{ &\frac{\theta}{[k_\tau^{SB}(\theta)]^2} C(\Delta_\theta)\frac{dk_\tau^{SB}}{d\tau} + \frac{\theta}{k_\tau^{SB}(\theta)} C'(\Delta_\theta)\frac{d\hat{q}}{dz}\left[\frac{dz}{d\tau} + \frac{dz}{dk}\frac{dk_\tau^{SB}}{d\tau}\right] \\ &- \kappa\frac{d\hat{q}}{dz}\left[\frac{dz}{d\tau} + \frac{dz}{dk}\frac{dk_\tau^{SB}}{d\tau}\right] - \rho\frac{dk_\tau^{SB}}{d\tau}\Bigg\} f(\theta)\mathrm{d}\theta, \end{aligned} \tag{A.36}$$

where $z = \tau k/\theta$ and $\Delta_\theta = q^B - \hat{q}(z)$. Using firms' optimization (10) and (11) allows us to rewrite the above expression as follows:

$$W'(\tau) = \int_{\underline{\theta}}^{\bar{\theta}} \Bigg\{ -(\tau-\kappa)\frac{1}{\theta C''(\Delta_\theta)}\left[k_\tau^{SB}(\theta) + \tau\frac{dk_\tau^{SB}}{d\tau}\right] + \left[\frac{\theta}{[k_\tau^{SB}(\theta)]^2}C(\Delta_\theta) - \rho\right]\frac{dk_\tau^{SB}}{d\tau}\Bigg\} f(\theta)\mathrm{d}\theta. \tag{A.37}$$

Note that the term in square brackets in the second line is strictly positive for all $\theta > \underline{\theta}$ by the definition of $k_\tau^{SB}(\theta)$ in (42), and equal to zero for $\theta = \underline{\theta}$. Moreover, for all (positive) $\tau \leq \kappa$, the first term in the integrand is weakly positive. Thus, $dk_\tau^{SB}(\theta)/d\tau > 0$ for all $\theta \in \Theta$ implies that $W'(\tau) > 0$ for all (positive) $\tau \leq \kappa$. Hence, the welfare-maximizing tax must exceed the Pigouvian level – the marginal environmental damage κ. □

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APPENDIX B. SUPPLEMENTARY MATERIAL (NOT FOR PUBLICATION)

B.1. **Calculations for the Example.** As Assumption 2 is satisfied by our leading example of cubic abatement costs, welfare is globally concave in (q,k). Hence, the welfare-optimal emission and capital levels are characterized by the first-order conditions

$$\frac{\theta}{k^*(\theta)}(q^B - q^*(\theta))^2 = \kappa, \qquad \frac{\theta}{3[k^*(\theta)]^2}(q^B - q^*(\theta))^3 = \rho. \tag{B.1}$$

Firms' individually optimal emissions $\hat{q}(\tau k/\theta)$ are straightforward. Firms' participation constraints are thus given by

$$V(\theta_i) = \frac{2}{3}\tau^{1.5}\sqrt{\frac{k(\theta)}{\theta}} - R(\theta) \geq 0. \tag{PC$_i$}$$

Under full information, both types' participation constraints bind. The bank maximizes

$$B(k(\theta)) = \frac{2}{3}\tau^{1.5}\sqrt{\frac{k(\theta)}{\theta}} - \rho k(\theta), \tag{B.2}$$

separately for each type, yielding

$$k_\tau^{FB}(\theta) = \frac{\tau^3}{9\theta\rho^2}. \tag{B.3}$$

Under asymmetric information, firms' incentive-compatibility constraints are

$$V(\theta_H) \geq V(\theta_L) - \frac{2}{3}\tau^{1.5}\sqrt{k_L}\frac{\sqrt{\theta_H}-\sqrt{\theta_L}}{\sqrt{\theta_H\theta_L}}, \tag{IC$_H$}$$

$$V(\theta_L) \geq V(\theta_H) + \frac{2}{3}\tau^{1.5}\sqrt{k_H}\frac{\sqrt{\theta_H}-\sqrt{\theta_L}}{\sqrt{\theta_H\theta_L}}. \tag{IC$_L$}$$

As (PC$_H$) and (IC$_L$) are binding, the bank solves

(B.4)

$$\max_{k_L \geq k_H} \; \nu\left[\frac{2}{3}\tau^{1.5}\sqrt{\frac{k_L}{\theta_L}} - \frac{2}{3}\tau^{1.5}\sqrt{k_H}\frac{\sqrt{\theta_H}-\sqrt{\theta_L}}{\sqrt{\theta_H\theta_L}} - \rho k_L\right] + (1-\nu)\left[\frac{2}{3}\tau^{1.5}\sqrt{\frac{k_H}{\theta_H}} - \rho k_H\right].$$

The first-order conditions yield the expressions for $k_\tau^{SB}(\theta_L)$ and $k_\tau^{SB}(\theta_H)$ stated in the main text. The Hessian for the bank's profit maximization problem is

$$\begin{pmatrix} \frac{\partial^2 B}{\partial k_L^2} & 0 \\ 0 & \frac{\partial^2 B}{\partial k_H^2} \end{pmatrix},$$

where

$$\frac{\partial^2 B}{\partial k_L^2} = -\frac{1}{6}\nu\left(\frac{\tau}{k_L}\right)^{1.5}\frac{1}{\sqrt{\theta_L}} < 0 \tag{B.5}$$

and

$$\frac{\partial^2 B}{\partial k_H^2} = -\frac{1}{6}(1-\nu)\left(\frac{\tau}{k_H}\right)^{1.5}\frac{\beta}{\sqrt{\theta_H}}. \tag{B.6}$$

Thus, the bank's objective is always globally concave in k_L, and it is globally concave in (k_L, k_H) if and only if $\beta \geq 0$. Hence, $k_\tau^{SB}(\theta_L)$ and $k_\tau^{SB}(\theta_H)$ as stated in the main text are the second-best profit-maximizing capital levels.

We derive the optimal tax-only policy. Welfare as a function of τ is given by

$$\begin{aligned} W(\tau) &= \nu \left[\Pi - \kappa q^B + \kappa \frac{\tau^2}{3\theta_L \rho} - \frac{2}{9} \frac{\tau^3}{\theta_L \rho} \right] \\ &+ (1-\nu) \left[\Pi - \kappa q^B + \kappa \frac{\tau^2 \beta}{3\theta_H \rho} - \frac{\beta(1+\beta)}{9} \frac{\tau^3}{\theta_H \rho} \right] \end{aligned} \tag{B.7}$$

The first-order condition is

$$W'(\tau) = \frac{\tau}{3\rho} \left[\frac{2\nu}{\theta_L}(\kappa - \tau) + \frac{(1-\nu)\beta}{\theta_H} \big(2\kappa - (1+\beta)\tau\big) \right] \overset{!}{=} 0. \tag{B.8}$$

The stationary point $\tau = 0$ is not welfare-maximizing, because

$$W''(0) = \frac{2\nu\kappa}{3\theta_L \rho} + \frac{2(1-\nu)\beta\kappa}{3\theta_H \rho} > 0, \tag{B.9}$$

(or, via an alternative argument, because there exists $\varepsilon > 0$ such that for all $\tau \in (0, \varepsilon)$, $W'(\tau) > 0$, and thus $W(\tau) > W(0)$). The interior stationary point yields τ^{**} as stated in the main text. The second-order condition holds:

$$\begin{aligned} W''(\tau^{**}) &= \nu \left[\frac{2\kappa}{3\theta_L \rho} - \frac{4\tau^{**}}{3\theta_L \rho} \right] \\ &+ (1-\nu) \left[\frac{2\beta\kappa}{3\theta_H \rho} - \frac{2\beta(1+\beta)\tau^{**}}{3\theta_H \rho} \right] < 0, \end{aligned} \tag{B.10}$$

because both expressions in squared brackets are negative. This implies that τ^{**} yields the welfare maximum.

We now turn to deriving the optimal policy mix when the regulator can introduce a tax and a uniform subsidy. With $\hat{\rho} := \rho - \sigma > 0$ replacing ρ, second-best capital levels are

$$k_{\tau,\sigma}^{SB}(\theta_L) = \frac{\tau^3}{9\theta_L \hat{\rho}^2} \quad \text{and} \quad k_{\tau,\sigma}^{SB}(\theta_H) = \frac{\tau^3}{9\theta_H \hat{\rho}^2} \beta^2. \tag{B.11}$$

Welfare can be written as

$$W(\tau, \sigma) = \Pi - \kappa q^B + \frac{\Lambda_1}{9\hat{\rho}} \big(3\kappa\tau^2 - \tau^3\big) - \frac{\rho \Lambda_2}{9\hat{\rho}^2} \tau^3, \tag{B.12}$$

where

$$\Lambda_1 := \frac{\nu}{\theta_L} + \frac{(1-\nu)\beta}{\theta_H}, \qquad \Lambda_2 := \frac{\nu}{\theta_L} + \frac{(1-\nu)\beta^2}{\theta_H}; \tag{B.13}$$

note that $\Lambda_1 > \Lambda_2$ for $\beta \in (0, 1)$, i.e., whenever a downward distortion in credit provision occurs. Note furthermore that the optimal tax-only policy can be written as

$$\tau^{**} = \kappa \cdot \frac{2\Lambda_1}{\Lambda_1 + \Lambda_2}. \tag{B.14}$$

Assuming for now an interior solution with $\tau > 0$ and $\sigma > 0$ (equivalently, $\hat{\rho} \in (0, \rho)$), the first-order conditions are

$$\frac{\partial W}{\partial \tau}(\tau, \sigma) = \frac{\tau}{3\hat{\rho}^2}\Big(2\Lambda_1 \hat{\rho}\, \kappa - \tau(\Lambda_1 \hat{\rho} + \Lambda_2 \rho)\Big) \overset{!}{=} 0, \tag{B.15}$$

$$\frac{\partial W}{\partial \sigma}(\tau, \sigma) = \frac{\tau^2}{9\hat{\rho}^3}\Big(3\Lambda_1 \hat{\rho}\, \kappa - \tau(\Lambda_1 \hat{\rho} + 2\Lambda_2 \rho)\Big) \overset{!}{=} 0. \tag{B.16}$$

Since $\tau > 0$ and $\hat{\rho} > 0$, the bracketed terms must be zero. Solving each condition for $\hat{\rho}$ yields

$$\hat{\rho} = \frac{\rho\, \Lambda_2\, \tau}{\Lambda_1(2\kappa - \tau)}, \tag{B.17}$$

$$\hat{\rho} = \frac{2\rho\, \Lambda_2\, \tau}{\Lambda_1(3\kappa - \tau)}. \tag{B.18}$$

Equating these two expressions implies $\tau^* = \kappa$. Plugging $\tau^* = \kappa$ into either expression for $\hat{\rho}$ and solving for σ gives

$$\sigma^* = \rho - \hat{\rho}^* = \rho\Big(1 - \frac{\Lambda_2}{\Lambda_1}\Big). \tag{B.19}$$

Substituting for Λ_1, Λ_2 yields the stated expression. At (κ, σ^*) the Hessian H satisfies

$$\frac{\partial^2 W}{\partial \tau^2}(\kappa, \sigma^*) = -\frac{2\Lambda_1^2}{3\Lambda_2}\frac{\kappa}{\rho} < 0, \qquad \det H(\kappa, \sigma^*) = \frac{\Lambda_1^6}{27\Lambda_2^4}\frac{\kappa^4}{\rho^4} > 0,$$

so the Hessian is negative definite. Due to

$$\frac{\partial^2 W}{\partial \tau^2}(0, \sigma) = \frac{2\Lambda_1 \kappa}{3\hat{\rho}} > 0, \tag{B.20}$$

$\tau = 0$ cannot constitute a maximum. Furthermore, $\sigma = 0$ cannot be optimal, because, using $\tau^{**} = 2\Lambda_1\kappa/(\Lambda_1 + \Lambda_2)$,

$$\frac{\partial W}{\partial \sigma}(\tau^{**}, 0) = \frac{(\tau^{**})^2}{9\rho^2} \cdot \Lambda_1 \kappa \cdot \frac{\Lambda_1 - \Lambda_2}{\Lambda_1 + \Lambda_2} \;>\; 0, \tag{B.21}$$

implying that (κ, σ^*) constitutes the optimal policy mix when a uniform subsidy is available.

By comparing the welfare-optimal capital levels $k^*(\theta_i)$ to the second-best capital levels $k_\tau^{SB}(\theta_i)$, it is immediate that the Pigouvian tax $\tau^* = \kappa$ and the tailored subsidies $\sigma_L^* = 0$ and $\sigma_H^* = \rho(1 - \beta)$ implement the welfare-optimal allocation.

We now adapt the cubic example to the continuum case by assuming that cost types are uniformly distributed on $[\underline{\theta}, \bar{\theta}]$. Since both the welfare-maximization problem and the bank's full-information problem are additively separable across types and involve no cross-type constraints, they can be solved point-wise. Hence, the welfare-optimal allocation and the full-information capital level are the same type-specific expressions as in the two-type example and are not derived again.

We now turn to asymmetric information. Recall that $V(\theta)$ denotes the net advantage obtained by type θ from accepting its designated contract. Under the cubic specification, mutual incentive compatibility for any pair of types $\theta_1 < \theta_2$ is equivalent to

$$\begin{aligned}\frac{2}{3}\tau^{3/2}\sqrt{k(\theta_2)}\left(\frac{1}{\sqrt{\theta_1}}-\frac{1}{\sqrt{\theta_2}}\right) &\leq V(\theta_1)-V(\theta_2)\\ &\leq \frac{2}{3}\tau^{3/2}\sqrt{k(\theta_1)}\left(\frac{1}{\sqrt{\theta_1}}-\frac{1}{\sqrt{\theta_2}}\right).\end{aligned} \tag{B.22}$$

As it satisfies Assumption 2, the cubic specification satisfies the single-crossing property, which can also be directly observed by noting that

$$\frac{2}{3}\tau^{3/2}\sqrt{k}\left(\frac{1}{\sqrt{\theta_1}}-\frac{1}{\sqrt{\theta_2}}\right)$$

is increasing in k. Consequently, the preceding inequalities require $k(\theta_1) \geq k(\theta_2)$, i.e., any globally incentive-compatible capital schedule must be non-increasing. Moreover, since the bank never finds it optimal to lend an infinite amount of capital, $k(\underline{\theta}) < \infty$. As the capital schedule is non-increasing, $k(\theta) \leq k(\underline{\theta})$ for all $\theta \in \Theta$. Noting that $\theta^{-1/2}$ is Lipschitz continuous on Θ with Lipschitz constant $1/(2\underline{\theta}^{3/2})$, the upper incentive-compatibility bound implies

$$|V(\theta_1)-V(\theta_2)| \leq \frac{\tau^{3/2}\sqrt{k(\underline{\theta})}}{3\underline{\theta}^{3/2}}|\theta_1-\theta_2|.$$

Hence, $V(\theta)$ is Lipschitz continuous and therefore absolutely continuous and differentiable almost everywhere. Fixing $\theta_1 < \theta_2$, dividing the pairwise incentive-compatibility inequalities by $\theta_2 - \theta_1$, and letting the two types approach one another yields, almost everywhere, the envelope condition

$$V'(\theta) = -\frac{1}{3}\left(\frac{\tau}{\theta}\right)^{3/2}\sqrt{k(\theta)}. \tag{B.23}$$

Conversely, for any non-increasing capital schedule and any absolutely continuous rent schedule $V(\theta)$ satisfying the envelope condition almost everywhere, the pairwise incentive-compatibility inequalities follow. For $\theta_1 < \theta_2$,

$$V(\theta_1)-V(\theta_2) = \int_{\theta_1}^{\theta_2}\frac{1}{3}\left(\frac{\tau}{s}\right)^{3/2}\sqrt{k(s)}\,\mathrm{d}s, \tag{B.24}$$

and monotonicity implies

$$\sqrt{k(\theta_2)} \leq \sqrt{k(s)} \leq \sqrt{k(\theta_1)} \qquad \text{for all } s \in [\theta_1,\theta_2].$$

Therefore,

$$\frac{2}{3}\tau^{3/2}\sqrt{k(\theta_2)}\left(\frac{1}{\sqrt{\theta_1}}-\frac{1}{\sqrt{\theta_2}}\right) \le V(\theta_1)-V(\theta_2) \le \frac{2}{3}\tau^{3/2}\sqrt{k(\theta_1)}\left(\frac{1}{\sqrt{\theta_1}}-\frac{1}{\sqrt{\theta_2}}\right).$$

Hence, under the cubic specification, global incentive compatibility is equivalent to a non-increasing capital schedule and an absolutely continuous rent schedule satisfying the envelope condition almost everywhere.

In the optimal contract, the participation constraint of the highest-cost type binds, so $V(\bar{\theta}) = 0$. Using the absolute continuity of $V(\theta)$, integrating the envelope condition (B.23) thus yields

$$V(\theta) = V(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} V'(s)\ \mathrm{d}s = \int_{\theta}^{\bar{\theta}} \frac{1}{3}\left(\frac{\tau}{s}\right)^{3/2}\sqrt{k(s)}\ \mathrm{d}s. \tag{B.25}$$

Since the integrand is non-negative, $V(\theta) \ge 0\ \forall\theta$, so all participation constraints are satisfied. Using the definition of $V(\theta)$ to substitute for $R(\theta)$, inserting the rent expression derived above, and changing the order of integration yields

$$B = \int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{2}{3}\tau^{3/2}\sqrt{\frac{k(\theta)}{\theta}} - \frac{1}{3}\tau^{3/2}\frac{\sqrt{k(\theta)}}{\theta^{3/2}}\frac{F(\theta)}{f(\theta)} - \rho k(\theta)\right] f(\theta)\ \mathrm{d}\theta. \tag{B.26}$$

Since types are uniformly distributed, $F(\theta) = (\theta-\underline{\theta})/(\bar{\theta}-\underline{\theta})$ and $f(\theta) = 1/(\bar{\theta}-\underline{\theta})$. Hence, the bank's profit becomes

$$B = \frac{1}{\bar{\theta}-\underline{\theta}}\int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{1}{3}\tau^{3/2}\sqrt{\frac{k(\theta)}{\theta}}\left(1+\frac{\underline{\theta}}{\theta}\right) - \rho k(\theta)\right] \mathrm{d}\theta. \tag{B.27}$$

The first-order condition yields the capital schedule stated in (45). Since the integrand is strictly concave in $k(\theta)$, this schedule uniquely maximizes the bank's objective when the monotonicity constraint is disregarded. The resulting schedule is strictly decreasing in the cost type θ and therefore satisfies the monotonicity constraint. Since it maximizes the bank's objective over the larger relaxed set and belongs to the set of non-increasing capital schedules, it also solves the reduced problem subject to monotonicity. Because that reduced problem is equivalent to the original screening problem, the corresponding rent and repayment schedules constitute a globally incentive-compatible mechanism, and $k_{\tau}^{SB}(\theta)$ as stated in (45) constitutes the second-best optimal capital schedule.

Before turning to the optimal tax-only policy, we confirm the stated properties of $\omega(h)$. For $h > 1$, the denominator of $\omega(h)$ is strictly positive. Moreover, for the numerator,

$$2\ln 1 - 1 + 1^{-2} = 0$$

and

$$\frac{d}{dh}\left(2\ln h - 1 + h^{-2}\right) = \frac{2(h^2-1)}{h^3} > 0 \qquad \text{for all } h > 1.$$

Hence, $\omega(h) > 0$ for all $h > 1$. Furthermore,

$$\begin{aligned}\omega(h) < \frac{1}{2} &\iff 2\ln h - 1 + h^{-2} < 2\left(\ln h + 1 - h^{-1}\right)\\ &\iff h^{-2} + 2h^{-1} - 3 < 0\\ &\iff \left(h^{-1}-1\right)\left(h^{-1}+3\right) < 0,\end{aligned}$$

which holds for all $h > 1$. Thus, $\omega(h) \in (0, 1/2)$. An application of l'Hôpital's rule yields

$$\lim_{h\to1^+}\omega(h) = \lim_{h\to1^+}\frac{2h^{-1}-2h^{-3}}{4\left(h^{-1}+h^{-2}\right)} = 0.$$

Finally, differentiating yields

$$\omega'(h) = \frac{(h+1)\left(3h^2 - 2h\ln h - 4h + 1\right)}{4h^4\left(\ln h + 1 - h^{-1}\right)^2}.$$

For $h > 1$, the denominator is strictly positive. Moreover, using $\ln h < h - 1$, we obtain

$$\begin{aligned}3h^2 - 2h\ln h - 4h + 1 &> 3h^2 - 2h(h-1) - 4h + 1\\ &= (h-1)^2 > 0.\end{aligned}$$

Hence, $\omega'(h) > 0$ for all $h > 1$.

We now derive the optimal tax-only policy. Welfare as a function of τ can be written as

(B.28)

$$W(\tau) = \Pi - \kappa q^B + \int_{\underline{\theta}}^{\bar{\theta}}\left[\frac{\kappa\tau^2}{6\rho\theta}\left(1+\frac{\underline{\theta}}{\theta}\right) - \frac{\tau^3}{18\rho\theta}\left(1+\frac{\underline{\theta}}{\theta}\right) - \frac{\tau^3}{36\rho\theta}\left(1+\frac{\underline{\theta}}{\theta}\right)^2\right] f(\theta)\ \mathrm{d}\theta.$$

Differentiating with respect to τ yields

$$W'(\tau) = \frac{1}{\rho}\int_{\underline{\theta}}^{\bar{\theta}}\left[\frac{\kappa\tau}{3\theta}\left(1+\frac{\underline{\theta}}{\theta}\right) - \frac{\tau^2}{6\theta}\left(1+\frac{\underline{\theta}}{\theta}\right) - \frac{\tau^2}{12\theta}\left(1+\frac{\underline{\theta}}{\theta}\right)^2\right] f(\theta)\ \mathrm{d}\theta. \tag{B.29}$$

For notational convenience, define

$$\Gamma_1 := \int_{\underline{\theta}}^{\bar{\theta}}\frac{1}{\theta}\left(1+\frac{\underline{\theta}}{\theta}\right) f(\theta)\ \mathrm{d}\theta = \frac{1}{\bar{\theta}-\underline{\theta}}\left[\ln h + 1 - h^{-1}\right], \tag{B.30}$$

$$\Gamma_2 := \int_{\underline{\theta}}^{\bar{\theta}}\frac{1}{\theta}\left(1+\frac{\underline{\theta}}{\theta}\right)^2 f(\theta)\ \mathrm{d}\theta = \frac{1}{\bar{\theta}-\underline{\theta}}\left[\ln h + \frac{5}{2} - 2h^{-1} - \frac{1}{2}h^{-2}\right]. \tag{B.31}$$

The derivative of welfare then becomes

$$W'(\tau) = \frac{\tau}{12\rho}\left[4\kappa\Gamma_1 - \tau(2\Gamma_1 + \Gamma_2)\right]. \tag{B.32}$$

The relevant non-zero stationary point therefore satisfies

$$\tau^{**} = \frac{4\Gamma_1}{2\Gamma_1 + \Gamma_2}\kappa = \kappa + \frac{2\Gamma_1 - \Gamma_2}{2\Gamma_1 + \Gamma_2}\kappa. \tag{B.33}$$

Because

$$\omega(h) = \frac{2\Gamma_1 - \Gamma_2}{2\Gamma_1}$$

and thus

$$\frac{2\Gamma_1 - \Gamma_2}{2\Gamma_1 + \Gamma_2} = \frac{\omega(h)}{2 - \omega(h)},$$

this yields the expression for τ^{**} stated in (46). It is immediate from (B.32) that welfare is increasing for $0 < \tau < \tau^{**}$ and decreasing for $\tau > \tau^{**}$. Hence, τ^{**} is the welfare-maximizing environmental tax.

Finally, we turn to the policy mix consisting of an environmental tax and a uniform loan subsidy. Let again $\hat{\rho} := \rho - \sigma > 0$ denote the bank's private unit cost of capital after the subsidy. Hence, second-best capital provision is now given by

$$k^{SB}_{\tau,\sigma}(\theta) = \frac{\tau^3}{36\,\hat{\rho}^2\,\theta}\left(1 + \frac{\underline{\theta}}{\theta}\right)^2. \tag{B.34}$$

Welfare as a function of both τ and σ can then be written as

$$W(\tau,\sigma) = \Pi - \kappa q^B + \frac{\kappa\tau^2}{6\hat{\rho}}\Gamma_1 - \frac{\tau^3}{18\hat{\rho}}\Gamma_1 - \frac{\rho\tau^3}{36\hat{\rho}^2}\Gamma_2. \tag{B.35}$$

Since $\partial W/\partial\sigma = -\partial W/\partial\hat{\rho}$, the first-order conditions can equivalently be stated in terms of τ and $\hat{\rho}$:

$$\frac{\partial W}{\partial \tau} = \frac{\tau}{12\hat{\rho}^2}\left[2\Gamma_1\hat{\rho}(2\kappa - \tau) - \Gamma_2\rho\tau\right] \stackrel{!}{=} 0, \tag{B.36}$$

$$\frac{\partial W}{\partial \hat{\rho}} = \frac{\tau^2}{18\hat{\rho}^3}\left[\Gamma_1\hat{\rho}(\tau - 3\kappa) + \Gamma_2\rho\tau\right] \stackrel{!}{=} 0. \tag{B.37}$$

For an interior solution with $\tau > 0$ and $\hat{\rho} \in (0,\rho)$, the bracketed terms must be zero. Adding the two bracketed equations yields

$$\Gamma_1\hat{\rho}(\kappa - \tau) = 0,$$

and hence $\tau^* = \kappa$. Substituting $\tau^* = \kappa$ into the first first-order condition gives

$$2\Gamma_1\hat{\rho}^*\kappa = \Gamma_2\rho\kappa,$$

so that

$$\hat{\rho}^* = \frac{\Gamma_2}{2\Gamma_1}\rho. \tag{B.38}$$

Thus, the optimal uniform subsidy is

$$\sigma^* = \rho - \hat{\rho}^* = \rho\left(1 - \frac{\Gamma_2}{2\Gamma_1}\right) = \rho\omega(h), \tag{B.39}$$

which is the expression stated in (47). It remains to verify that this stationary point is the welfare maximum. For any fixed $\hat{\rho} > 0$, the derivative $\partial W/\partial\tau$ is positive below and negative above

$$\tau(\hat{\rho}) = \frac{4\Gamma_1\hat{\rho}}{2\Gamma_1\hat{\rho} + \Gamma_2\rho}\kappa. \tag{B.40}$$

Hence, $\tau(\hat{\rho})$ is the unique welfare-maximizing tax conditional on $\hat{\rho}$. We can therefore reduce the problem to maximizing

$$\bar{W}(\hat{\rho}) := W(\tau(\hat{\rho}), \hat{\rho}).$$

By the chain rule and the first-order condition defining $\tau(\hat{\rho})$,

$$\bar{W}'(\hat{\rho}) = \underbrace{\frac{\partial W}{\partial\tau}(\tau(\hat{\rho}), \hat{\rho})}_{=0}\tau'(\hat{\rho}) + \frac{\partial W}{\partial\hat{\rho}}(\tau(\hat{\rho}), \hat{\rho}) = \frac{\partial W}{\partial\hat{\rho}}(\tau(\hat{\rho}), \hat{\rho}).$$

Substituting $\tau(\hat{\rho})$ into $\partial W/\partial\hat{\rho}$ yields

$$\bar{W}'(\hat{\rho}) = \frac{8\Gamma_1^3\kappa^3\left(\Gamma_2\rho - 2\Gamma_1\hat{\rho}\right)}{9\left(2\Gamma_1\hat{\rho} + \Gamma_2\rho\right)^3}. \tag{B.41}$$

This expression is positive for $\hat{\rho} < \hat{\rho}^*$ and negative for $\hat{\rho} > \hat{\rho}^*$. Therefore, welfare is maximized at $\hat{\rho}^* = \Gamma_2\rho/(2\Gamma_1)$ and $\tau^* = \kappa$. Since $\omega(h) \in (0, 1/2)$, the corresponding subsidy satisfies $\sigma^* \in (0, \rho)$.

B.2. **Supplementary Derivations for the Continuous Cost Types Case.** In this appendix, we discuss sufficient conditions under which the second-best optimal capital schedule $k_\tau^{SB}(\theta)$ is strictly decreasing in the type θ and strictly increasing in the environmental tax τ. Throughout this subsection, we assume that $C(\cdot)$ is three times continuously differentiable on $(0, q^B)$.

Let

$$u(k, \tau, \theta) = -\tau\hat{q}\left(\frac{\tau k}{\theta}\right) - \frac{\theta}{k}C\left(q^B - \hat{q}\left(\frac{\tau k}{\theta}\right)\right). \tag{B.42}$$

Moreover, let

$$b(k, \tau, \theta) = \tau q^B + u(k, \tau, \theta) + \partial_\theta u(k, \tau, \theta)M(\theta) - \rho k, \tag{B.43}$$

where

$$M(\theta) := F(\theta)/f(\theta).$$

First, we impose the standard assumption regarding the Mills ratio.

Assumption 3. *The Mills ratio $M(\theta)$ is non-decreasing; i.e., $M'(\theta) \geq 0$.*

Second, we assume that the bank's objective function $b(\cdot)$ exhibits regularity properties.

Assumption 4. *The function $b(k,\tau,\theta)$ is strictly concave in k and has a unique positive maximizer $\tilde{k}(\tau,\theta) > 0$ for any $\theta \in \Theta$ and $\tau > 0$; i.e., $\partial^2_{kk} b(k,\tau,\theta) < 0$.*

The function $\tilde{k}(\tau,\theta)$ solves the first-order condition

$$\partial_k u(k,\tau,\theta) + \partial^2_{k\theta} u(k,\tau,\theta) M(\theta) = \rho. \tag{B.44}$$

By the implicit function theorem, we obtain

$$\frac{\partial \tilde{k}}{\partial \theta} = -\frac{[1+M'(\theta)]\partial^2_{k\theta} u(k,\tau,\theta) + M(\theta)\partial^3_{k\theta\theta} u(k,\tau,\theta)}{\partial^2_{kk} b(k,\tau,\theta)}, \tag{B.45}$$

$$\frac{\partial \tilde{k}}{\partial \tau} = -\frac{\partial^2_{k\tau} u(k,\tau,\theta) + M(\theta)\partial^3_{k\theta\tau} u(k,\tau,\theta)}{\partial^2_{kk} b(k,\tau,\theta)}. \tag{B.46}$$

First, we consider the sign of $\partial \tilde{k}/\partial\theta$. The denominator is strictly negative by Assumption 4. Regarding the numerator, note that

$$\partial^2_{k\theta} u(k,\tau,\theta) = \frac{1}{k^2}\left[C(\Delta) - \frac{\tau k}{\theta}\frac{C'(\Delta)}{C''(\Delta)}\right] < 0, \tag{B.47}$$

and

$$\partial^3_{k\theta\theta} u(k,\tau,\theta) = \frac{\tau}{k\theta^2}\frac{C'(\Delta)}{C''(\Delta)}\left[1 - \frac{C'(\Delta)C'''(\Delta)}{[C''(\Delta)]^2}\right], \tag{B.48}$$

where $\Delta = q^B - \hat{q}(z)$ and $z = \tau k/\theta$. The sign of the derivative $\partial^3_{k\theta\theta} u(k,\tau,\theta)$ is generally undetermined. However, as long as $\partial^3_{k\theta\theta} u(k,\tau,\theta)$ is not sufficiently positive to outweigh the first term in the numerator of (B.45), the numerator is strictly negative and, hence, $\partial \tilde{k}/\partial\theta < 0$. This is the case, for instance, in our leading example with cubic abatement costs and a uniform type distribution.

Now consider the sign of the derivative $\partial \tilde{k}/\partial\tau$. Again, the denominator is strictly negative by Assumption 4. Regarding the numerator, note that

$$\partial^2_{k\tau} u(k,\tau,\theta) = \frac{1}{k}\frac{C'(\Delta)}{C''(\Delta)} > 0, \tag{B.49}$$

and

$$\partial^3_{k\tau\theta} u(k,\tau,\theta) = -\frac{\tau}{\theta^2 C''(\Delta)}\left[1 - \frac{C'(\Delta)C'''(\Delta)}{[C''(\Delta)]^2}\right]. \tag{B.50}$$

The sign of $\partial^3_{k\tau\theta} u(k,\tau,\theta)$ is generally undetermined. However, as long as $\partial^3_{k\tau\theta} u(k,\tau,\theta)$ is not sufficiently negative to outweigh the first term in the numerator of (B.46), the numerator is strictly positive and, hence, $\partial \tilde{k}/\partial\tau > 0$.

Finally, note that if the monotonicity constraint is indeed non-binding, then $k_\tau^{SB}(\theta) = \tilde{k}(\tau,\theta)$ for all $\theta \in \Theta$.

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